Quantifying time-varying uncertainty and risk for the real price of oil

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Disclaimer: The views expressed herein are solely those of the presenter and do not necessarily reflect the views of Norges Bank. Background is paper: **Quantifying time-varying uncertainty and risk for the real price of oil**. Joint work with **Knut Are Aasveit and Jamie Cross** from Norges Bank and BI Norwegian Business School

General motivation

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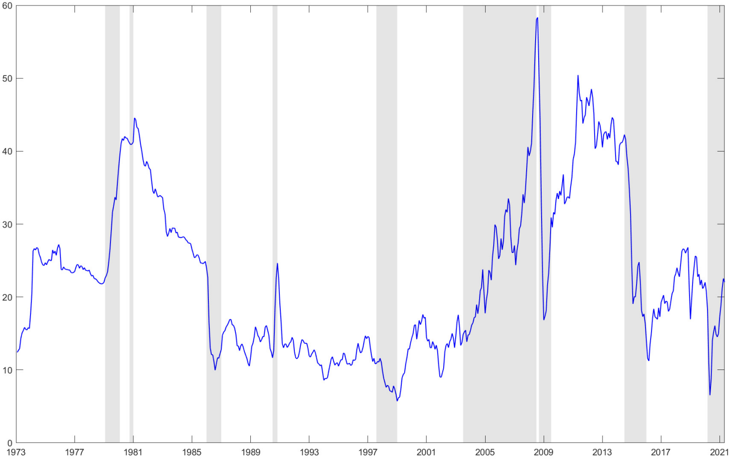
  - **Central banks, private sector forecasters and international organizations**
- But also crucial for how some sectors operate their business
  - **Airlines, utilities and automobile manufacturers**
- ...But the **price of oil is not easy to forecast**

Specific Motivation 1: Given the changing data pattern in the real price of oil which is subjected to shocks and volatility, how to forecast it ?



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- It is widely accepted to either use the **current spot price or the price of oil futures contracts** as the forecast of the price of oil.

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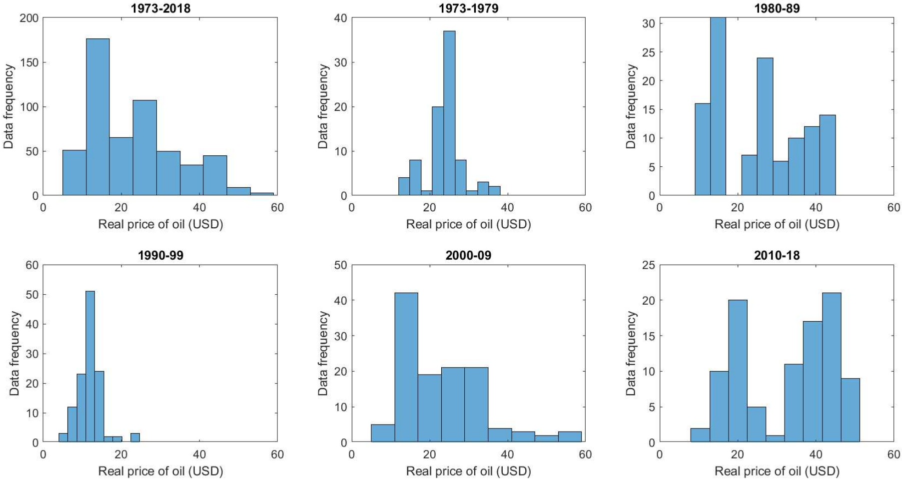
- These papers focus on evaluating **point forests** and find that
  - It's **hard to beat a random walk** in out-of-sample oil price forecasting exercises
  - But careful attention to the **economic fundamentals** that are driving energy markets can lead to practical improvements in forecasts

Specific Motivation 2: How to model and forecast the changing distribution of real price of oil ?



Promising approach: Combine information from different sources

Basic practice of combining information in macroeconomic and financial forecasting is to make use of a **weighted combination** of **forecasts** from many sources, say **experts, models and/or large micro-data sets.** Let y_t be the variable of interest, and let $\tilde{y}_{1t}, \ldots, \tilde{y}_{n,t}$ be forecasted values from $i = 1, ..., n$ models, with weights $w_{1t}, \ldots, w_{n,t}$ where n maybe small (from a committee of experts) or large (from a large micro-data set). Then, basic practice is to make use of:

$$y_t = \sum_{i=1}^{n} w_{it} \tilde{y}_{it} \tag{1}$$

where $\tilde{y}_{it}$ should be a good approximation to y_t.

- **Problem with Practice** Many agencies handle this averaging and updating informally.

Our approach in combining information from different sources

- **Challenge** Give this practice a **Bayesian probabilistic foundation** in order to evaluate **practical** issues as follows: Make use of **Forecast Density Combinations (FDC)** features which, given information on data en model specification allows the evaluation of **Conditional Probabilities of (extreme) events: Recession probability; Turning point probability; Probabilistic warnings about defaults and crises in macroeconomics and finance; Value-at-Risk etc etc.**

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- **Fast growth in Big Data** gives opportunity of more accurate forecast measures. Analogy with weather forecasting using many satellite pictures. But in economics issue like multimodality, skewness, diverging ratio's of, for instance, government expenditures and GDP are not trivial and time-varying.

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- **New Tool:Parallel Computing: New Hardware and Software** give openings to solve complex problems. **Machine learning with several hidden layers using neural networks have a direct connection with filtering methods in nonlinear time series models**.

Relation to literature on combining probabilistic forecasts

- Combining forecast densities using **weighted linear combinations** of prediction models, evaluated using various scoring rules
  - Hall and Mitchell (2007); Amisano and Giacomini (2007); Jore et al. (2010); Hoogerheide et al. (2010); Kascha and Ravazzolo (2010); Geweke and Amisano (2011, 2012); Gneiting and Ranjan (2013); Aastveit et al. (2014)

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- **Complex combination approaches that allows for time-varying weights with possibly both learning and model set incompleteness**
  - Koop and Korobilis (2012); Billio et al. (2013); Casarin et al. (2015); Pettenuzzo and Ravazzolo (2016); Del Negro et al. (2016); Aastveit et al. (2018); McAlinn and West (2019); McAlinn et al. (2020); Takanashi and McAlinn (2020); Casarin et al. (2020)

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- **No studies on how to quantify forecast uncertainty associated with the dynamic behaviour of the real price of crude oil.**

Plan of talk

- **Theoretical contributions: Structure of Forecast Density Combination (FDC): Probability Model, Equation System and Algorithm**
- Choice of model set for our empirical application
- **Empirical Contributions: Working of FDC in measuring time-varying uncertainty and risk in the real price of oil**
- Conclusion

Theoretical Contributions: Summary

- **Flexible Bayesian Forecast Density Combination** allows for cross-section and time dependent **Bayesian weight learning** and **Diagnostic learning about model incompleteness**. This **self-learning** is closely related to **Machine-Learning**.

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- **Flexible Bayesian Forecast Density Combination** allows for cross-section and time dependent **Bayesian weight learning** and **Diagnostic learning about model incompleteness**. This **self-learning** is closely related to **Machine-Learning**.
- **Model Representation and Efficient Computation** Model is a **Generalised Linear State Space Model which allows the use of numerically efficient standard Markov Chain Monte Carlo simulation methods**. **Filtering** methods from State Space models are directly connected to the **integration of the hidden layers** in machine learning.

Structure of our FDC: Probability Model

- Let $\tilde{\boldsymbol{y}}_{\boldsymbol{t}}' = (\tilde{y}_{1t}, \ldots, \tilde{y}_{nt})$ be the forecasted values from $i = 1, ..., n$ models for the variable of interest y_t. In a simulation context $\tilde{y}_{it}$ **is a draw from the forecast distribution of model** M_i **with density** $p(\tilde{y}_{it}|I_{it-1}, M_i)$ and data set I_{it-1}.

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- Let $\boldsymbol{v}_{\boldsymbol{t}}{}' = (v_{0t}, v_{1t}, \ldots, v_{nt})$ **be latent continuous random variable parameters which are used to weight the different forecasts and combine these forecasts**

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- Let $\boldsymbol{v_t}' = (v_{0t}, v_{1t}, \ldots, v_{nt})$ **be latent continuous random variable parameters which are used to weight the different forecasts and combine these forecasts**
- The decomposition of the joint density of $(y_t, \boldsymbol{v}_t, \tilde{\boldsymbol{y}}_t)$ for the case of continuous random variables is:

$$p(y_t|I_{t-1}, M) = \int\int p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_{\boldsymbol{t}})p(\boldsymbol{v}_t|\tilde{\boldsymbol{y}}_{\boldsymbol{t}})p(\tilde{\boldsymbol{y}}_{\boldsymbol{t}}|I_{t-1}, M)d\boldsymbol{v}_t d\tilde{\boldsymbol{y}}_{\boldsymbol{t}}, \qquad (2)$$

where I_{t-1} is the joint information set of all models and M the union of all models. The integrals are of dimension n and $n+1$ for each time observation.

Structure of our FDC: Choice of the different densities

A key step is to give content to the different densities.

- $p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_t)$ is labeled the **multivariate normal combination density** :

$$p(y_t|\boldsymbol{v}_t, \tilde{\boldsymbol{y}}_t) = n(y_t|v_{0t} + \sum_{i=1}^{n} v_{it}\tilde{y}_{it}, \sigma_t^2), \tag{3}$$

  where time-varying constant v_{0t} in the conditional mean allows for forecast adjustments to shocks and regime changes in the data. σ_t^2 allows for time-varying volatility.

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- $p(\boldsymbol{v}_t|\tilde{\boldsymbol{y}}_t)$ is labeled the **density of latent time-varying parameter weights** and specified as:

$$p(\boldsymbol{v}_t|\boldsymbol{v}_{t-1}, \boldsymbol{\Sigma}_t) = n(\boldsymbol{v}_t|\boldsymbol{v}_{t-1}, \boldsymbol{\Sigma}_t), \qquad (4)$$

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- $p(\tilde{\boldsymbol{y}}_t|I_{t-1}, M)$ is labeled **the joint forecast density of the different models.** Due to the conditional independence assumption it is given as:

$$p(\tilde{\boldsymbol{y}}_t|I_{t-1}, M) = \prod_{i=1}^{n} p(\tilde{y}_{it}|I_{i(t-1)}, M_i). \tag{5}$$

Structure of our FDC: The corresponding equation system is an econometric interpretation of Bayesian Predictive Synthesis approach from Mike West et al

- **The Equation System:** a **multivariate regression model** with **generated regressors** $\tilde{y}_t$, given as draws from the forecast distributions of the different models and **time-varying parameters** v_{it} draws:

$$y_t = v_{0t} + \sum_{i=1}^{n} v_{it} \tilde{y}_{it} + \varepsilon_t :: \varepsilon_t \sim NID(0, \sigma_t^2), t = 1, \ldots, T. \quad (6)$$

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$$v_{it} = v_{it-1} + \varepsilon_{vt} :: \varepsilon_{vt} \sim NID(0, \sigma_{vt}^2 = \sigma_t^2 w_t), i = 0, \ldots, n. \quad (7)$$

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- Mike West et al calls this a DLM (Dynamic Linear Factor Model).

Learning from errors: Forecast errors and model set incompleteness

- The disturbance ε_t implied by the combination density is given as:

$$\varepsilon_t = y_t - (v_{0t} + \sum_{i=1}^{n} v_{it}\tilde{y}_{it}). \tag{8}$$

It is a **weighted** combination of forecast errors: $y_t - \tilde{y}_{it}, i = \ldots n$.
Forecast errors are due to:

  - **Sudden shocks in the series, volatility**
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- The dynamic behaviour of the individual disturbance ε_{it} from model M_i given as:

$$\varepsilon_{it} = y_t - (v_{0,it} + v_{it}\tilde{y}_{it}), \tag{9}$$

which indicates the **weighted** forecast error in the i-the model.

Road Map of the Probability model as Generalised Linear State Space System

Time series models

$M_i \ (i = 1, \ldots, n)$:

$\tilde{y}_{it} \sim p(\tilde{y}_{it} | I_{i(t-1)}, M_i)$

Stochastic volatility model:

$\varepsilon_t \sim NID(0, \sigma_t^2)$

$\sigma_t^2 = \frac{\delta \sigma_{t-1}^2}{\gamma_t}$

$\gamma_t \sim \text{Beta}(\frac{\delta h_{t-1}}{2}, \frac{(1-\delta) h_{t-1}}{2})$

$h_t = \delta h_{t-1} + 1$

Central equation:

$y_t = v_{0t} + \sum_{i=1}^n v_{it} \tilde{y}_{it} + \varepsilon_t$

Random walk learning for

unrestricted latent variables:

$v_{it} = v_{i(t-1} + \varepsilon_{vt}$

$\varepsilon_{vt} \overset{iid}{\sim} \mathcal{N}(0, \sigma_{vt}^2 = \sigma_t^2 w_t)$

$w_t = \frac{\beta - 1}{\beta} w_{t-1}$

Outline of Algorithm

3 stage Markov Chain Monte Carlo

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1. **Forecasting proceeds as follows:**
2. Given a generated $v_{it}, i = 1, \ldots, n$, a generated SV value, a generated $\tilde{y}_{it}, i = 1, \ldots, n$ and using (6) generate a one step predicted value y_{t+1}.

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3. Repeating this process gives a synthetic sample of future values and a forecast density at time $t+1$.
4. Very Important feature from this MCMC procedure: **The uncertainty in the generated forecasts from the different models is directly carried forward in the uncertainty of the combined forecast density**. In contrast, **frequentists methods use a two-step method and they suffer from the generated regressor problem.**

Individual models

General framework for constructing forecast densities from individual models

- General stochastic volatility model with Student's t-distributed errors given by

$$S_{t+h|t} - \hat{S}_{t+h|t} = \varepsilon_{t+h|t}, \quad \varepsilon_{t+h|t} \sim T(\mu, e^{h_{t+h|t}}, \nu), \tag{10}$$

$$h_{t+h|t} = \mu + \phi(h_{t+h-1|t} - \mu) + \zeta_{t+h|t}, \quad \zeta_{t+h|t} \sim NID(0, \omega^2), \tag{11}$$

in which $|\phi| < 1$ and $\hat{S}_{t+h|t}$ is a point forecast of the real price.

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$$S_{t+h|t} - \hat{S}_{t+h|t} = \varepsilon_{t+h|t}, \quad \varepsilon_{t+h|t} \sim T(\mu, e^{h_t+h|t}, \nu), \tag{10}$$
$$h_{t+h|t} = \mu + \phi(h_{t+h-1|t} - \mu) + \zeta_{t+h|t}, \quad \zeta_{t+h|t} \sim NID(0, \omega^2), \tag{11}$$

in which $|\phi| < 1$ and $\hat{S}_{t+h|t}$ is a point forecast of the real price.

- Obtain draws from the forecast distribution of $\tilde{S}_{t+h|t}$, conditional on the model estimates

$$\tilde{S}_{t+h|t} = \hat{S}_{t+h|t} + \hat{\varepsilon}_{t+h|t}, \quad \varepsilon_t \sim T(0, e^{\hat{h}_{t+h|t}}, \hat{\nu}), \tag{12}$$

in which $\hat{\varepsilon}_{t+h|t}, \hat{h}_{t+h|t}$ and $\hat{\nu}$ are posterior draws from the estimated stochastic volatility model.

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- **Futures & West Texas Intermediate (WTI) oil futures prices (Futures)**

$$\hat{S}_{t+h|t} = S_{t|t}(1 + f_t^{WTI,h} - s_t^{\text{WTI}} - \mathbb{E}_t[\pi_{t+h}^{(h)}]), \tag{15}$$

Individual forecasting models

- **Spread & Spread Between the Spot Prices of Gasoline and Crude Oil (Spread)**

$$\hat{S}_{t+h|t} = S_{t|t}\exp(\hat{\beta}[s_t^{\text{gas}} - s_t^{\text{WTI}}] - \mathbb{E}_t[\pi_{t+h}^{(h)}]), \tag{16}$$

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- **Oil market Vector Autoregression (VAR)**

$$y_t = b + \sum_{i=1}^{p} B_i y_{t-i} + e_t, \tag{18}$$

Empirical contributions

Empirical work

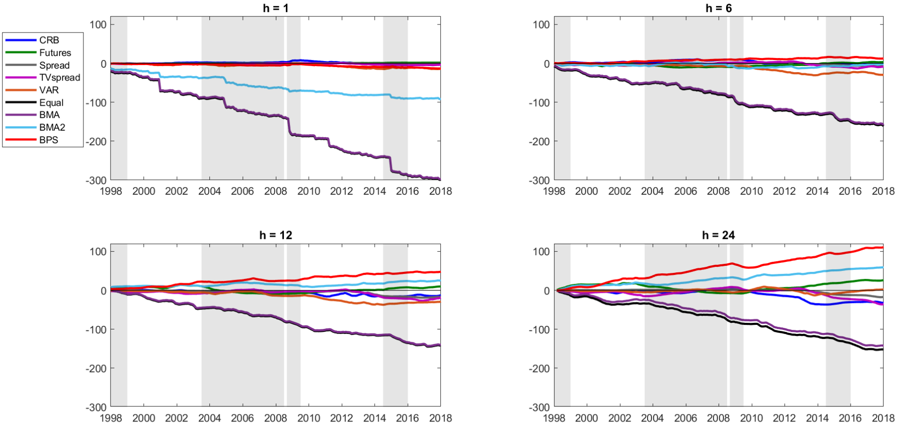
- Forecast monthly real price of crude oil
  - Real-time data as in Baumeister and Kilian (2012, 2015)
  - Training sample: 1992:01-1998:02
  - Evaluation sample: 1998:03-2017:12
  - Forecast evaluation: Root Mean Squared Forecast Error (RMSFE), Log Predictive Score (LPS) and their time behaviour, Time behaviour of weights and diagnostic measures.
  - Forecast horizons: $h = 1$, $h = 6$, $h = 12$, $h = 24$
- Consider different model combinations
  - BPS, BMA, BMA with rolling window weights, and equal weights

Density and point forecast results relative to a no-change benchmark, Evaluation sample 1998:03-2017:12

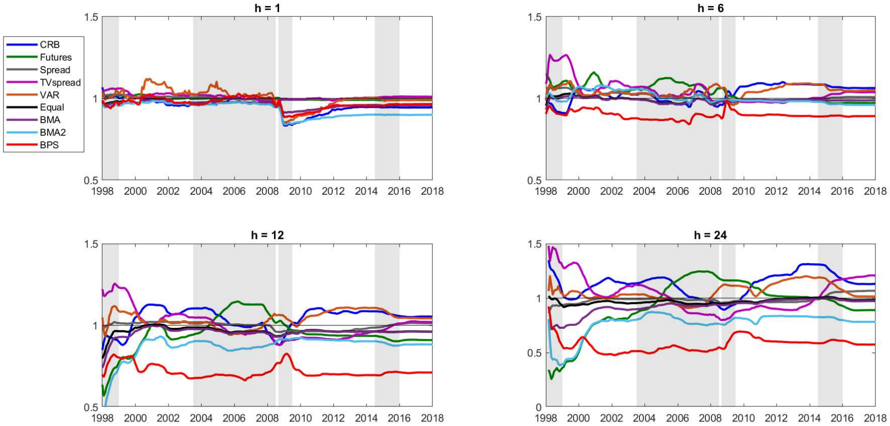
				LPS					
Horizon	CRB	Futures	Spread	TVspread	VAR	Equal	BMA	BMA2	BPS
1	0,46	**1,69***	-0,55	-3,62	-14,26	-298,95**	-297,04**	-91,44**	-13,23
6	-0,21	3,11	-5,10*	-8,14	-29,57**	-161,18**	-158,22**	-2,55*	**12,59****
12	-12,50	10,02*	-16,56**	-19,14**	-28,74**	-141,46**	-139,63**	25,85*	**47,73****
24	-32,48**	26,10**	-16,91**	-35,25**	2,34	-152,15**	-142,21**	59,14**	**110,96****

				RMSFE					
Horizon	CRB	Futures	Spread	TVspread	VAR	Equal	BMA	BMA2	BPS
1	0,95	0,99*	1,00	1,01	0,99	0,96*	0,96*	**0,90***	0,97
6	1,06*	0,97*	1,01	1,04	1,05*	0,99	0,99	0,96**	**0,89****
12	1,05	0,91**	1,01	1,02	1,04	0,96**	0,96**	0,88**	**0,71****
24	1,13**	0,89**	1,07	1,21**	1,01**	0,98	0,97**	0,78**	**0,57****

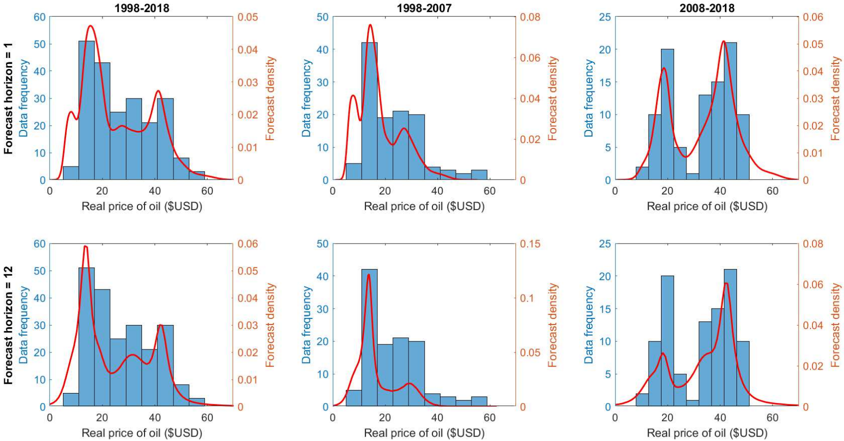
Time patterns of forecast means of cumulative Log Predictive Scores relative to a no-change model benchmark



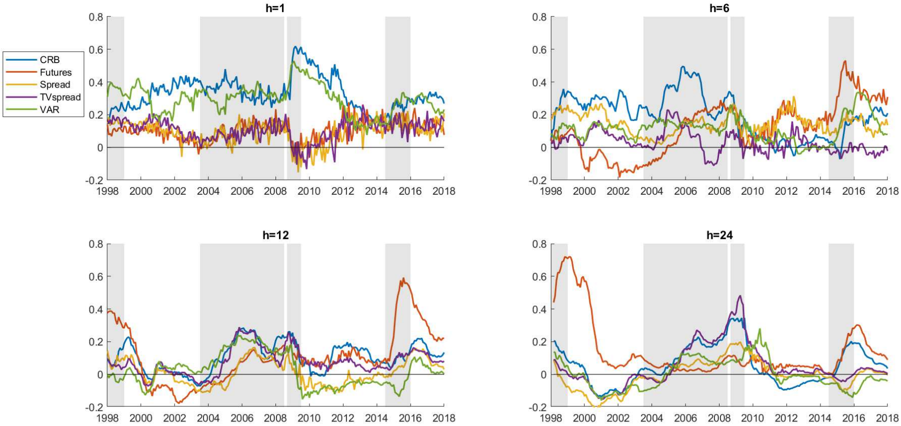
Time patterns of forecast means of Root Mean Squared Forecast Errors relative to a no-change model benchmark



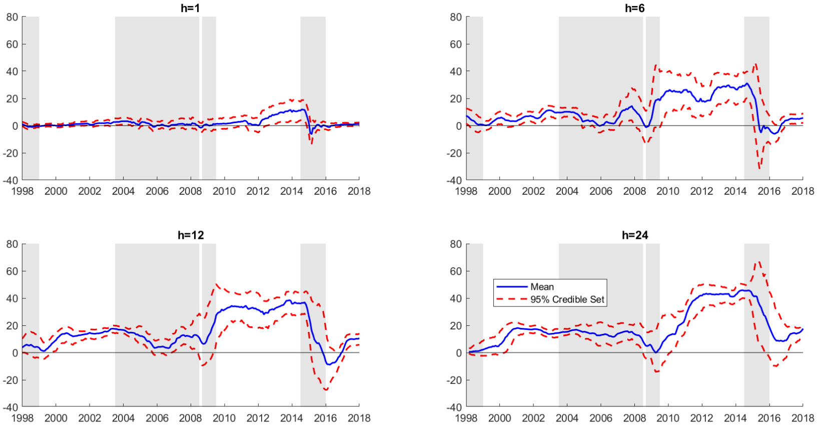
Observed data densities and estimated forecast density combinations pooled over specific subperiods



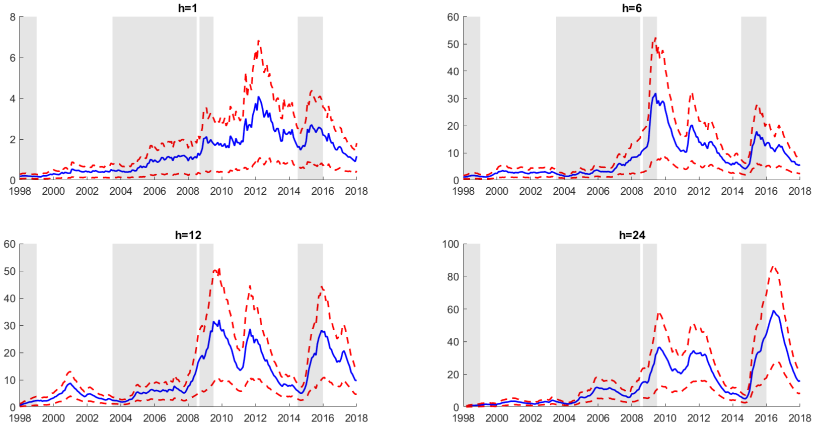
Time patterns of forecast means of model weights (v_{it}) in the FDC model based on BPS



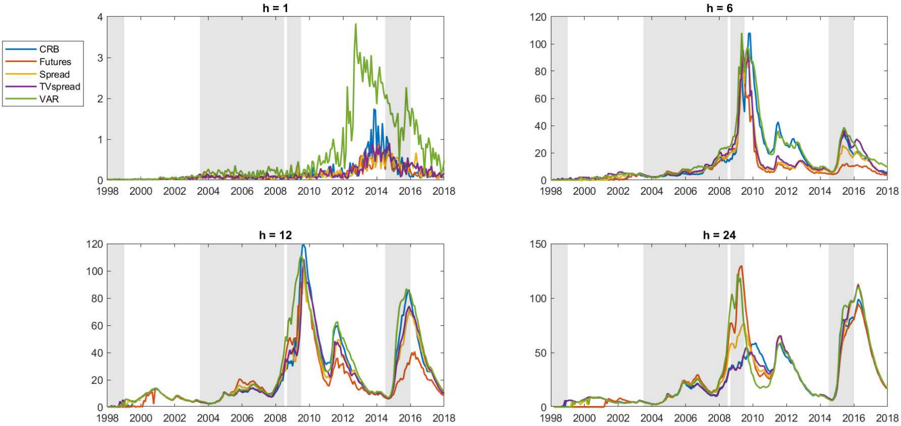
Time pattern of forecast means of intercept (v_{0t}) in the FDC model based on BPS



Time pattern of forecast means of variance (σ_t^2) for the central equation in BPS model.



Time patterns of forecast means of variances (σ^2_{it}) for individual models in the central equation in BPS model



Robustness checks

- Robustness check: **Alternative oil price series** We focused on forecasting the IRAC price of crude oil, which is commonly viewed as a proxy for the global price of oil. Two alternative series that are frequently cited in the press are the Brent and West Texas Intermediate (WTI) prices of crude oil.

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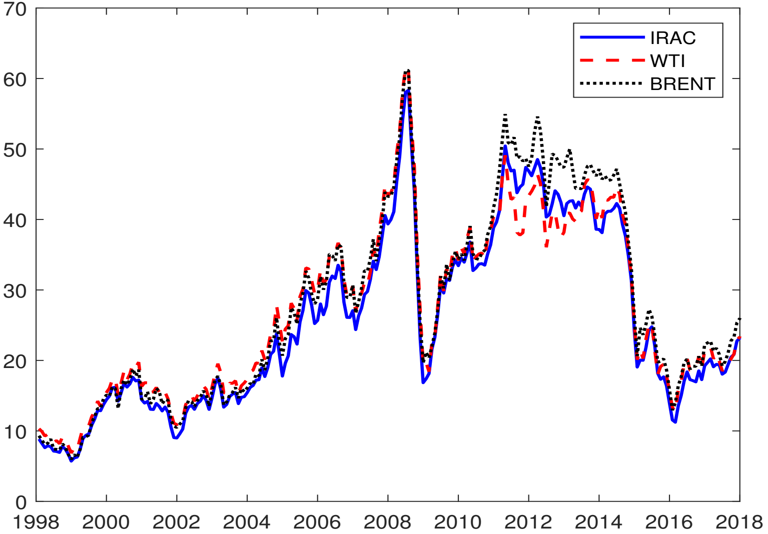
- Robustness check: **Alternative model sets** We estimated alternative models, see Table earlier. The main insight is that none of the additional models increase the forecast accuracy of the main BPS specification.
- Robustness check: **Alternative weighting procedures** Are time-varying parameter models as important as allowing for time-varying combination weights for our data set? In our data set time-varying combination weights in the main BPS specification are more important (give more accuracy) than individual time-varying parameter models.

Robustness checks: Alternative oil price series



Summary of results

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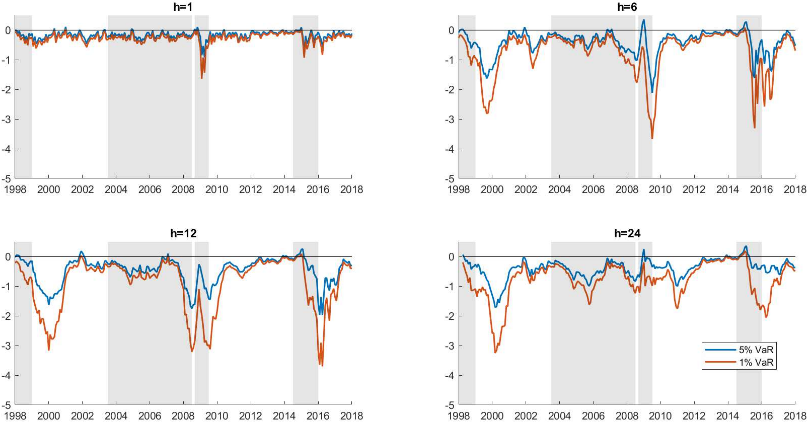
4. Our combination is robust to model set incompleteness and misspecification
   - Time-varying intercept component that can adapt during episodes of low frequency signals
   - Built-in diagnostic information measures about forecast inaccuracy and/or model set incompleteness

Risk analysis

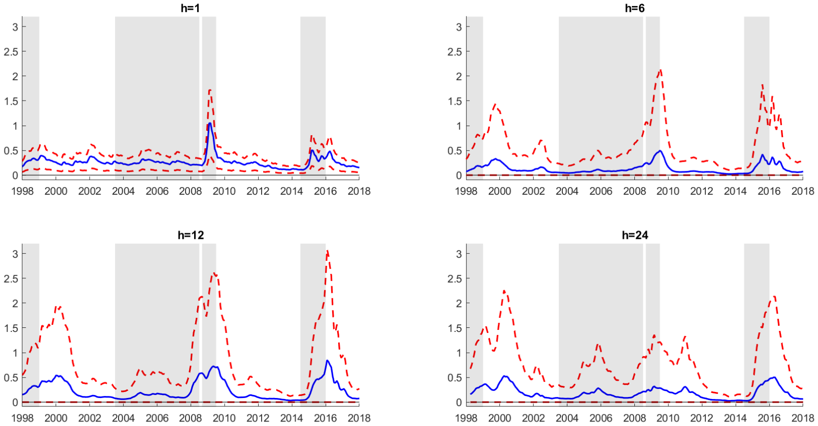
- Analyze the **risk and return** properties of investing in the global market for oil using our BPS modelling approach as an investment tool
  - Profit and loss distribution
  - Value-at-Risk (VaR)
  - Minimum Variance Hedge (MVH)

Profit and loss distribution (profit positive and loss negative) over the forecast evaluation period

Value at Risk for BPS over the forecast evaluation period



Minimum Variance Hedge ratios over the forecast evaluation period



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  1. **Time-varying and self-learning combination weights.**
  2. Explicitly modelling and estimation of time-varying forecast biases and facets of miscalibration of individual forecast densities and time-varying inter-dependencies among models
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  3. Provide a **diagnostic learning analysis of model set incompleteness and learn from previous forecast mistakes.**
- We have provided an extensive set of empirical results about time-varying forecast uncertainty and risk for the real price of oil

Topics for Further Research

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- **Policy**. Applications to **Policy Issues**: In the field of Finance using Decision Models. Related paper by me and co-authors in *Journal of Econometrics 2019* is on **Forecast Density Combinations of Dynamic Models and Data Driven Portfolio Strategies** based on the concept of momentum patterns in the financial return data using US industrial portfolios 1929-2015. **Challenge to do this for macro-models**.

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- **More efficient parallel computing.**

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