Correlation stress testing of stock and credit portfolios

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joint work with Fabian Woebbeking

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Overview

Motivation for correlation stress-testing

London Whale

Background Correlation parameterisation Stress testing correlations Scenario selection and reverse stress testing Results

General approach

Bayesian factor selection Preliminary results

Motivation

- Principal idea: link economically meaningful scenarios to correlation scenarios
- **Stress testing**: portfolio effect of adverse correlation scenario
- Reverse stress testing: identify worst-case scenarios and their impact
- First application: correlation stress testing of "London Whale" portfolio

Packham, N. and Woebbeking, F.: A factor-model approach for correlation scenarios and correlation stress-testing. Journal of Banking and Finance, 101 (2019), 92-103. [Ink

Work in progress: generalisation to credit and stock portfolios

Regulatory aspects

- EU / Basel-regularion (CRR = Capital Requirments Regulation):
 - CRR Article 386(1)(g):
 "[..]institution shall frequently conduct a rigorous programme of stress testing, including reverse stress tests[..]"
 - CRR Article 375(1):
 "[..]potential for significant basis risks in hedging strategies[..]"
 - CRR Article 376(3)(b):

"[..] assess [..] internal model, particularly with regard to the treatment of concentrations."

- CRR Article 377:

"Requirements for an internal model for correlation trading"

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The "London Whale"

- "London Whale": 2012 Loss at JPMorgan Chase & Co. of approx.
 6.2 bn USD on a credit derivatives portfolio.
- Authorised trading position, hence risk management problem.
- Synthetic credit portfolio (SCP): portfolio of credit index derivatives
- Approx. 120 positions, CDX and iTraxx index and tranche products, both investment grade and high-yield.
- Publicy available information: JPMorgan, 2013; United-States-Senate, 2013a,b

The "London Whale" strategy

- "Smart short" strategy: credit protection on high yield is financed by selling protection on investment grade indices.
- ▶ End of 2011: decision to reduce SCP's risk-weighted assets (RWA's).
- Avoid liquidation costs by increasing positions with opposite market sensitivity (hedges).
- 23 March 2012: Senior executives ordered to stop trading on SCP; net notional of 157 bn USD (up 260% from September 2011).
- Risk management of SCP focussed on value-at-risk (VaR) and CSW-10 (credit spread widening of 10 basis points), but ignored correlation changes.

The "London Whale" PnL



Figure: Cumulative PnL of the SCP in USD (2012). Data source: JPMorgan (2013).

The "London Whale" positions

Table: Top 10 Positions of SCP, 23 March 2012, USD net notional; several positions have a market share close to 50%.

	Ir	ndex			
Name	Series	Tenor	Tranche (%)	Protection	Net Notional (\$)
CDX.IG	9	10yr	Untranched	Seller	72,772,508,000
	9	7yr	Untranched	Seller	32,783,985,000
	9	5yr	Untranched	Buyer	31,675,380,000
iTraxx.EU	9	5yr	Untranched	Seller	23,944,939,583
	9	10yr	22 - 100	Seller	21,083,785,713
	16	5yr	Untranched	Seller	19,220,289,557
CDX.IG	16	5yr	Untranched	Buyer	18,478,750,000
	9	10yr	30 - 100	Seller	18,132,248,430
	15	5yr	Untranched	Buyer	17,520,500,000
iTraxx.EU	9	10yr	Untranched	Seller	17,254,807,398
Net Total					137,517,933,681

Data source: United-States-Senate (2013a, Exhibit 36) and DTCC (2014, Section 1, Table 7). London Whale

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Interest-rate modelling: Correlation parameterisation

- Parametric correlation model in interest-rate modelling / LIBOR market model, e.g. Rebonato (2002); Brigo (2002); Schoenmakers and Coffey (2000); Packham (2005)
- Simplest case: Correlation c_{ij} between two forward LIBOR's is given by

$$c_{ij} = e^{-\beta|i-j|},$$

where $\beta > 0$ is a parameter, and i, j represent maturities.

 Captures stylised fact that correlations decay with increasing maturity difference

Correlation parameterisation

- Idea: For other asset classes (e.g. equity, credit) adapt parameterisation with suitable risk factors such as geographic regions, industries, investment grade vs. high-yield, ...
- $C: n \times n$ -correlation matrix of n financial instruments' returns.
- ► Factors that determine the correlations: x = (x¹,...,x^m)'.
- Correlation of securities i and j modelled as

$$c_{ij} = \exp(-(\beta_1 |x_i^1 - x_j^1| + \beta_2 |x_i^2 - x_j^2| + \dots + \beta_m |x_i^m - x_j^m|),$$

$$i, j = 1, \dots, n,$$

with β_1, \ldots, β_m positive coefficients, determined through calibration.

• Functional form implies that the greater "distance" $|x_i^k - x_j^k|$, the greater de-correlation amongst securities *i* and *j*.

► If two instruments are identical in all respects, then correlation is 1. London Whale

Correlation parameterisation

- Given historical asset returns, parameters β₁,..., β_m are determined e.g. by OLS on transformed correlations - ln(c_{ij}).
- Scenario (e.g. "the correlation between investment grade and high-yield securities decreases") is implemented by increasing corresponding β-parameter.
- With parameters calibrated on a regular basis, the parameter history can be used to obtain reasonable scenarios.

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Stress-testing correlations

- **Stress-test**: Effect on portfolio due to an adverse scenario.
- A shift in correlation has no *instantaneous* effect on portfolio value, therefore consider **portfolio risk**.
- Portfolio risk measured by value-at-risk (VaR) in variance-covariance approach:

$$\mathsf{VaR}_{\alpha} = -V_0 \cdot \mathrm{N}_{1-\alpha} \cdot \left(\mathbf{w}^{\mathsf{T}} \, \boldsymbol{\Sigma} \, \mathbf{w}\right)^{1/2},$$

with

- current position value V_0 ,
- $N_{1-\alpha}:\;(1-\alpha)\mbox{-quantile}$ of the standard normal distribution,
- vector of portfolio weights \boldsymbol{w} and
- covariance matrix Σ .
- ► For correlation stress test, need to consider portfolio variance

$$\mathbf{w}^{\intercal} \mathbf{\Sigma} \mathbf{w}$$

Core and peripheral risk factors

- Following e.g. Kupiec (1998), stress scenario comprises
 - "core" risk factors (the ones that are stressed)
 - "peripheral" risk factors (affected by stress).
- ▶ β_s : j < m core factor parameters that are stressed directly
- β_u : remaining m j peripheral risk factor parameters
- In normal distribution setting, optimal estimator of Δβ_u conditional on Δβ_s:

$$\mathbb{E}(\Delta \boldsymbol{\beta}_u | \Delta \boldsymbol{\beta}_s) = \Sigma_{us} \Sigma_{ss}^{-1} \Delta \boldsymbol{\beta}_s,$$

where Σ_{us} and Σ_{ss} denote the covariance and variance matrices of β_u and β_s .

Joint stress test of correlation and volatility

- Correlation shocks often coincide with volatility shocks, see e.g. (Alexander and Sheedy, 2008; Longin and Solnik, 2001; Loretan and English, 2000).
- Simple model that combines both: **multivariate** *t*-**distribution**.
- In this case *d*-dimensional vector of asset returns X follows a normal variance mixture distribution with decomposition (e.g. Ch. 6.2 of McNeil *et al.* (2015))

 $\mathbf{X} = \sqrt{V} \cdot A \cdot \mathbf{Z},$

where – $\mathbf{Z} \sim \mathrm{N}(0, I_k)$,

- V is a scalar r.v. independent of \mathbf{Z} ,
- $V \sim \log(1/2\nu, 1/2\nu)$, i.e., V follows an inverse gamma distribution,
- A is a $d \times k$ matrix such that $\tilde{\Sigma} = AA^T$.

Example: stress-testing a homogeneous portfolio*

- Homogeneous portfolio with n = 2^m assets exhibiting all 2^m combinations of binary correlation risk factors.
- Securities all have equal volatility σ , and the portfolio is equally-weighted, $w = \frac{1}{n}$.

Example: stress-testing a homogeneous portfolio*

Proposition

The portfolio variance is given by

$$\mathbf{w}^{\mathsf{T}} \boldsymbol{\Sigma} \mathbf{w} = \frac{\sigma^2}{n} \prod_{k=1}^{m} \left(1 + e^{-\beta_k} \right).$$

Proposition

The portfolio variance when j of the β -risk factors coefficients are stressed by $\Delta\beta$ is given by

$$\mathbf{w}^{\mathsf{T}} \, \boldsymbol{\Sigma} \, \mathbf{w} = \frac{\sigma^2}{n} \left(1 + e^{-(\beta + \Delta\beta)} \right)^j \cdot \left(1 + e^{-\left(\beta + \frac{j \cdot \rho_\beta}{(j-1)\rho_\beta + 1} \Delta\beta\right)} \right)^{m-j}$$

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Scenario selection and Mahalanobis distance

- Scenario selection: What is the worst scenario amongst all scenarios that occur within some pre-given range?
- Let β = (β₁,...,β_m)^T be a random vector with E(β) = β
 and covariance matrix Σ_β.
- Mahalabonis distance:

$$D(\boldsymbol{\beta}) = \left((\boldsymbol{\beta} - \overline{\boldsymbol{\beta}})^{\mathsf{T}} \boldsymbol{\Sigma}_{\boldsymbol{\beta}}^{-1} (\boldsymbol{\beta} - \overline{\boldsymbol{\beta}}) \right)^{1/2}.$$

Maha determines normal probabilities; multivariate normal density:

$$\mathbf{n}(\mathbf{x}) = \mathsf{det}(2\pi\Sigma)^{-\frac{1}{2}} \exp\left(-\frac{1}{2}(\mathbf{x}-\mu)^T\Sigma^{-1}(\mathbf{x}-\mu)\right).$$

> This generalises to elliptical distributions.

Illustration of Mahalabonis distance in bivariate case

- 10,000 simulations; shown are the 50% simulated pairs with smallest Maha.
- If β ~ N(β,Σ_β), then D² ~ χ²(m), i.e., Maha² follows a χ²-distribution (see histogram and density).

$$\boldsymbol{\beta}^* = \operatorname*{argmax}_{\boldsymbol{\beta}:D^2(\boldsymbol{\beta}) \leq h} \mathsf{VaR}_{\alpha}(\boldsymbol{\beta}),$$

with correlation matrix imposed by β and h a quantile of the $\chi^2(m)$ -distribution.



Proposition

In the homogeneous setting, the risk factor coefficients of the worst scenario within a given Mahalanobis distance \sqrt{h} are constant, i.e., $\beta_1^* = \cdots = \beta_m^* = \beta^*$, and given by

$$\beta^* = \overline{\beta} - \sqrt{\frac{h\sigma_{\beta}^2(1 + (m-1)\rho_{\beta})}{m}}.$$

Analytical results for non-homogeneous case also available.

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Risk factors and correlation model

- All calculations on SCP portfolio of 23 March 2012 (117 instruments).
- Risk factors: CDX vs. itraxx
 - investment grade vs. high yield
 - maturity
 - index series
 - index vs. tranche
- Parameterised correlation matrix:

$$\begin{split} c_{ij} &= \exp\left(-(\beta_1|\mathsf{isCDX}_i - \mathsf{isCDX}_j| + \beta_2|\mathsf{isIG}_i - \mathsf{isIG}_j| + \beta_3|\mathsf{maturity}_i - \mathsf{maturity}_j| \\ &+ \beta_4|\mathsf{series}_i - \mathsf{series}_j| + \beta_5|\mathsf{isIndex}_i - \mathsf{isIndex}_j|)\right). \end{split}$$

Calibration and results

- At any point in time, β₁,..., β₅ calibrated from credit spread returns of 250 preceding days.
- ► Time period: 1 March 2011 12 April 2012. Data source: Markit
- Example: calibrated coefficients on for 23 March 2012: $\beta = \begin{pmatrix} 0.35 & 0.37 & 0.21 & 0.05 & 0.20 \end{pmatrix}'$
- Strong de-correlation amongst CDX vs. itraxx, investment grade vs. high-yield

Calibration and results



Figure: Correlation matrices of 23 March 2012. Left: Empirical correlation matrix; right: parameterised (complete) correlation matrix. The dark red entries are unavailable correlations due to insufficient data. The three blocks of highly correlated data consist of (from top to bottom): CDX IG, CDX HY and iTraxx securities.

Calibration and results



Figure: Coefficients associated with correlation parameterisation of CDX and itraxx positions in London Whale position; 01/03/2011-12/04/2012. All distances are normalised to [0, 1] to make the coefficients comparable. Data source: Markit.

	(correlation st	ress	joint	stress
Quantile	$VaR_{0.99}$	<i>t</i> -VaR _{0.99}	Change(%)	<i>t</i> -VaR _{0.99}	Change(%)
base case	339.32	354.98		354.98	
0.7	366.87	383.80	8.12	386.28	8.82
0.8	369.39	386.44	8.86	416.41	17.31
0.9	372.89	390.10	9.89	464.40	30.83
0.95	375.76	393.11	10.74	510.54	43.82
0.99	381.08	398.67	12.31	617.38	73.92
0.995	383.00	400.68	12.87	664.73	87.26
0.999	386.88	404.74	14.02	780.37	119.84
$unconstrained^*$	620.96	649.62	83.00	1252.53	252.85

Risk implications from correlation stress-testing

*Unconstrained w.r.t. correlation changes; $\tilde{\alpha}$ remains on the 0.999 level.

Table: SCP portfolio's 1-day 99% value-at-risk for different Mahalanobis quantile constraints. Percentage changes denote relative distance to base VaR. For joint stress, percentage changes refer to base *t*-VaR scenario. Parameter ν is calibrated to 13.5. Volatility stress level $\tilde{\alpha}$ for joint stress test is set to quantile in column one.

Risk-driver identification (reverse stress test)



Figure: Box-plots of correlation parameters.

Dots: observed parameters as of 23.03.2012.

Crosses: worst-case scenario under a 99%-quantile Mahalanobis distance.

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Factor selection

- Risk factors in "London Whale" were tailored to specific portfolio.
- In practice, factor models comprising industries and countries as factors are often used to model the dependencies of asset returns.
- Problem: How to assign factors to assets?
- Number of factors should be small, but include all important factors.
- Some prior information is typically available: country of firm's headquarter, primary industry
- Use Bayesian variable selection to determine small number of factors driving asset return

Link correlations to risk factors

• Association of asset $i \in \{1, \dots, p\}$ with factor $k \in \{1, \dots, d\}$:

 ${f 1}_{\{k,i\}}$

Correlation parameterisation:

$$c_{ij} = \tanh\left(\eta + \underbrace{\sum_{k=1}^{d} \lambda_k |\mathbf{1}_{\{k,i\}} - \mathbf{1}_{\{k,j\}}|}_{\text{"inter"-correlations}} + \underbrace{\sum_{k=1}^{d} \nu_k \mathbf{1}_{\{k,i\}} \mathbf{1}_{\{k,j\}}}_{\text{"intra"-correlations}}\right),$$

with coefficients $\eta, \lambda_1, \ldots, \lambda_d, \nu_1, \ldots, \nu_d \in \mathbb{R}$.

General approach

Link correlations to risk factors

- $tanh : \mathbb{R} \to [-1, 1]$ allows for negative correlations.
- ► tanh used in inferential statistics on sample correlation coefficients (~> Fisher transformation).
- The following summation formula is helpful for a rough interpretation of the coefficients:

$$\tanh(x+y) = \frac{\tanh x + \tanh y}{1 + \tanh x \tanh y}$$



General approach

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Bayesian factor selection

Bayesian variable selection

- Different methods, e.g.
 - Bayesian model selection compares posterior probabilities of different models.
 - Spike and slab priors include an indicator variable for each coefficient and determines the indicator variable's posterior probability of taking value one.
- ▶ In our setting, **Bayesian model selection** worked best.

Bayesian model selection

- Denote candidate models by M_i , $i = 1, \ldots, m$.
- In a linear regression setting, each model M_i includes a specific subset of independent variables and excludes the other variables.
- Posterior model probability:

 $p(M_i|\boldsymbol{y}) \propto p(\boldsymbol{y}|M_i)p(M_i),$

where

- $p(M_i)$ is the prior model probability
- $p(\boldsymbol{y}|M_i)$ is called the marginal likelihood.

(see e.g. Appendix B.5.4 of (Fahrmeir et al., 2013))

Bayesian model comparison

▶ For each model, define indicator variables γ_k , k = 1, ..., p, with

$$\gamma_k = \begin{cases} 1, & \text{ if } \beta_k \neq 0 \\ 0, & \text{ else.} \end{cases}$$

Posterior inclusion probabilities (PIP):

$$\mathbf{P}(\gamma_k = 1 | \boldsymbol{y}) = \sum_{\beta_k \in M_{\gamma}} \mathbf{P}(M_{\gamma} | \boldsymbol{y}).$$
(1)

- If number of parameters p is large, then full calculation of 2^p posterior model probabilities is infeasible.
- \Rightarrow Use Monte Carlo simulation or MCMC.
- ► PIP of *k*-th factor determined as **frequency** of visited models with $\gamma_k = 1$ relative to the total number of visited models.

General approach

Example: VW

- We model VW stock returns as a linear function of MSCI (GICS) industry and MSCI country factors.
- The data set consists of daily returns of
 - MSCI stock indices representing 11 industries and 24 countries;
 - individual stock returns (DAX and S&P 500 names)
- ► Time period: 2002-2018

Example: VW

▶ Factors with PIP greater 0.5 are selected:

>>>	<pre>>> print(res[res['PIP']>0.5].round(4))</pre>			
	coef	PIP	BVS	pvalue
4	MXWOOCD Index	1.0000	1.0000	0.0000
9	MXWOOTC Index	0.9848	0.9900	0.0017
10	MXWOOUT Index	0.9996	1.0000	0.0000
18	MSDUSZ	0.6788	0.4940	0.0105
19	MSDUAT	0.7998	0.7613	0.0000
34	MSDUGR	1.0000	1.0000	0.0000

- Here, CD (Consumer Discretionary) and GR (Germany) have prior inclusion probability of 1.
- The other prior inclusion probabilities are chosen to include eight factors on average.

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Factor selection

- ► Factors: MSCI stock indices representing 11 industries and 24 countries
- Individual stocks: 500 S&P constituents, 30 DAX constituents
- Daily data from 2002-2018 (Source: Bloomberg, MSCI)
- Factor assignment re-calibrated every quarter, based on 3-years of daily data (52 quarters)
- Prior: hard-code primary country and industry; include 6 factors on expectation
- Following graphs show the number of quarters certain factors were assigned for selected stocks.

Factor selection (Apple and Citibank)



General approach

Factor selection (VW and Deutsche Bank)



General approach

Empirical vs. fitted correlations

- Left: empirical; right: fitted
- Based on 250 day estimation window, 2 July 2018
- Bottom: block of German (DAX) assets



Distributions of fitted parameters

Only "inter"-correlations are included, "intra"-correlations excluded



Distributions of fitted parameters





Distributions of fitted parameters

- Some factor loadings
- Left: "inter"-factor loading only
- Right: including "intra"-factor loadings (constant omitted)



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Conclusion

- We develop a correlation stress testing framework, linking (risk) factors with correlations.
- Reverse stress tests can be conducted by assigning the factor loading a distribution.
- "London whale": a significant de-correlation between investment grade and high yield credit derivatives broke the "hedges" in the SCP.
- Simple correlation stress testing exposes the significant risks in a portfolio with high notional and low RWA.
- General case: factors (e.g. industries, countries) are linked firms via Bayesian variable selection methods
- Outlook: apply PCA to generate factors; factors can often be given an economic interpretation (global factor, Europe, etc.)

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Thank you!

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Regulatory view and aspects

CRR Article 386(1)(g):

"the institution shall frequently conduct a rigorous programme of stress testing, including reverse stress tests, which encompasses any internal model used for purposes of this Chapter. [..]"

Regulatory view and aspects <a>Back

CRR Article 375(1):

"[..] Hedging or diversification effects associated with long and short positions involving different instruments or different securities of the same obligor, as well as long and short positions in different issuers, may only be recognised by explicitly modelling gross long and short positions in the different instruments. Institutions shall reflect the impact of material risks that could occur during the interval between the hedge's maturity and the liquidity horizon as well as the potential for significant basis risks in hedging strategies by product, seniority in the capital structure, internal or external rating, maturity, vintage and other differences in the instruments. [..]"

Regulatory view and aspects

CRR Article 376(3)(b):

"perform a variety of stress tests, including sensitivity analysis and scenario analysis, to assess the qualitative and quantitative reasonableness of the internal model, particularly with regard to the treatment of concentrations. Such tests shall not be limited to the range of events experienced historically [..]"

CRR 377: "Requirements for an internal model for correlation trading"

Regulatory view and aspects

ECB guide to internal models, Market Risk, 150 (Link):

"In accordance with Article 376(3)(b) of the CRR, institutions must perform sensitivity analysis and scenario analysis to assess the qualitative and quantitative reasonableness of the internal model, particularly with regard to the treatment of concentrations. [..]

In particular, the ECB considers it best practice that this sensitivity analysis includes, as a minimum, the following basic analysis, where systematic risk factor weights or correlations of risk factors in the model are shifted up or down by a fixed value or set to generic values: [..]

(e) all correlations between systematic factors are set to 100% (weights of issuers to their respective systematic factors remain unchanged);
(f) all correlations between systematic factors are set to 0% (weights of issuers to their respective systematic factors remain unchanged).
[..]