MODEL-BASED OPTIMIZATION FOR EXPENSIVE BLACK-BOX PROBLEMS AND HYPERPARAMETER OPTIMIZATION

Bernd Bischl

Computational Statistics, LMU Munich

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SEQUENTIAL MODEL-BASED OPTIMIZATION

PARALLEL BATCH PROPOSALS

Multicriteria SMBO

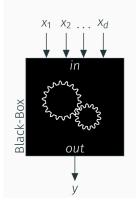
INTERESTING CHALLENGES

ML MODEL SELECTION AND HYPERPARAMETER OPTIMIZATION

Section 1

SEQUENTIAL MODEL-BASED OPTIMIZATION

EXPENSIVE BLACK-BOX OPTIMIZATION



$$y = f(\mathbf{x}) , \quad f : \mathbb{X} \to \mathbb{R}$$
 (1)

$$\mathbf{x}^* = \operatorname*{arg\,min}_{\mathbf{x} \in \mathbb{X}} f(\mathbf{x})$$
 (2)

- ► y, target value
- ▶ $x \in \mathbb{X} \subset \mathbb{R}^d$, domain
- ► f(x) function with considerably long runtime
- ▶ Goal: Find optimum x^{*}

SEQUENTIAL MODEL-BASED OPTIMIZATION

- Setting: Expensive black-box problem $f : x \to \mathbb{R} = min!$
- Classical problem: Computer simulation with a bunch of control parameters and performance output; or algorithmic performance on 1 or more problem instances; we often optimize ML pipelines
- ► Idea: Let's approximate *f* via regression!

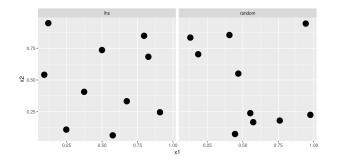
GENERIC MBO PSEUDO CODE

- Create initial space filling design and evaluate with f
- ► In each iteration:
 - Fit regression model on all evaluated points to predict f̂(x) and uncertainty ŝ(x)
 - Propose point via infill criterion

$$\mathsf{El}(x)\uparrow \Longleftrightarrow \hat{f}(x)\downarrow \land \hat{s}(x)\uparrow$$

- Evaluate proposed point and add to design
- EGO proposes kriging (aka Gaussian Process) and El Jones 1998, Efficient Global Opt. of Exp. Black-Box Functions

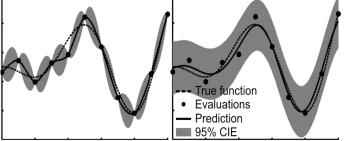
LATIN HYPERCUBE DESIGNS



- ► Initial design to train first regression model
- ► Not too small, not too large
- ▶ LHS / maximin designs: Min dist between points is maximized
- But: Type of design usually has not the largest effect on MBO, and unequal distances between points could even be beneficial

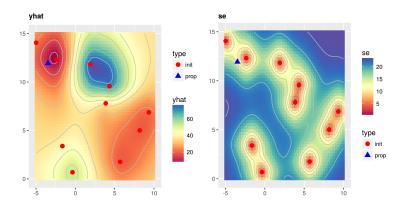
KRIGING AND LOCAL UNCERTAINTY PREDICTION Model: Zero-mean GP Y(x) with const. trend and cov. kernel $k_{\theta}(x_1, x_2)$.

►
$$\mathbf{y} = (y_1, \dots, y_n)^T$$
, $\mathbf{K} = (k(\mathbf{x}_i, \mathbf{x}_j))_{i,j=1,\dots,n}$
► $\mathbf{k}_*(\mathbf{x}) = (k(\mathbf{x}_1, \mathbf{x}), \dots, k(\mathbf{x}_n, \mathbf{x}))^T$
► $\hat{\mu} = \mathbf{1}^T \mathbf{K}^{-1} \mathbf{y} / \mathbf{1}^T \mathbf{K}^{-1} \mathbf{1}$ (BLUE)
► Prediction: $\hat{f}(\mathbf{x}) = E[Y(x)|Y(x_i) = y_i, i = 1, \dots, n] = \hat{\mu} + \mathbf{k}_n(x)^T \mathbf{K}^{-1}(\mathbf{y} - \hat{\mu}\mathbf{1})$
► Uncertainty: $\hat{s}^2(\mathbf{x}) = Var[Y(x)|Y(x_i) = y_i, i = 1, \dots, n] = \sigma^2 - \mathbf{k}_n^T(x) \mathbf{K}^{-1} \mathbf{k}_n(x) + \frac{(1 - \mathbf{1}^T \mathbf{K}^{-1} \mathbf{k}_n^T(x))^2}{\mathbf{1}^T \mathbf{K}^{-1} \mathbf{1}}$



Kriging / GP is a spatial model

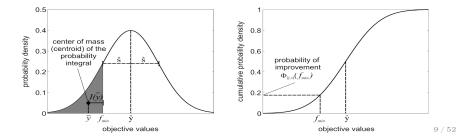
- ► Correlation between outcomes (y₁, y₂) depends on dist of x₁, x₂ E.g. Gaussian covar kernel k(x₁, x₂) = exp(-||x₁-x₂||)/2σ
- Useful smoothness assumption for optimization
- ▶ Posterior uncertainty at new *x* increases with dist to design points
- Allows to enforce exploration



INFILL CRITERIA: EXPECTED IMPROVEMENT

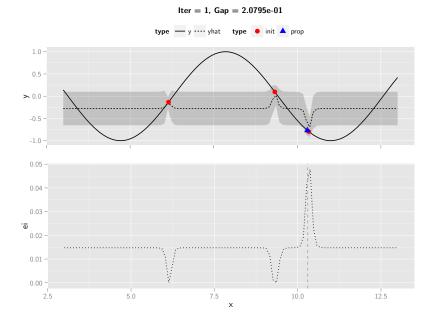
- ▶ Define improvement at x over best visited point with y = f_{min} as random variable I(x) = |f_{min} Y(x)|⁺
- For kriging $Y(x) \sim N(\hat{f}(x), \hat{s}^2(x))$ (given x = x)
- Now define EI(x) = E[I(x)|x = x]
- Expectation is integral over normal density starting at f_{min}
- Alternative: Lower confidence bound (LCB) $\hat{f}(\mathbf{x}) \lambda \hat{s}(\mathbf{x})$

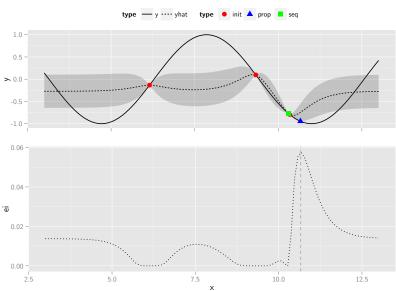
Result:
$$EI(\mathbf{x}) = \left(f_{min} - \hat{f}(\mathbf{x})\right) \Phi\left(\frac{f_{min} - \hat{f}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right) + \hat{s}(\mathbf{x})\phi\left(\frac{f_{min} - \hat{f}(\mathbf{x})}{\hat{s}(\mathbf{x})}\right)$$

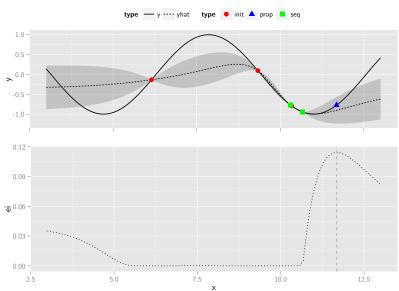


FOCUSSEARCH

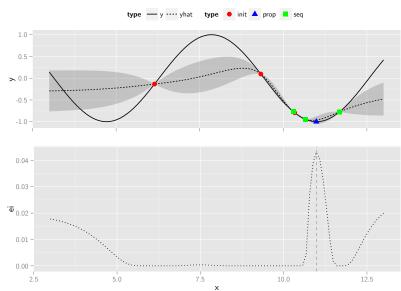
- ► El optimization is multimodal and not that simple
- But objective is now cheap to evaluate
- Many different algorithms exist, from gradient-based methods with restarts to evolutionary algorithms
- ► We use an iterated, focusing random search coined "focus search"
- ► In each iteration a random search is performed
- ► We then shrink the constraints of the feasible region towards the best point in the current iteration (focusing) and iterate, to enforce local convergence
- ► Whole process is restarted a few times
- Works also for categorical and hierarchical params



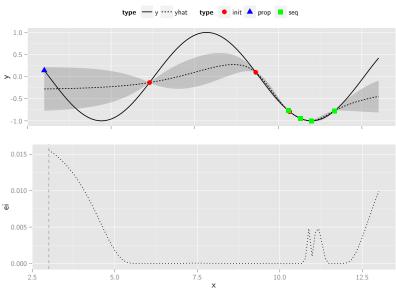




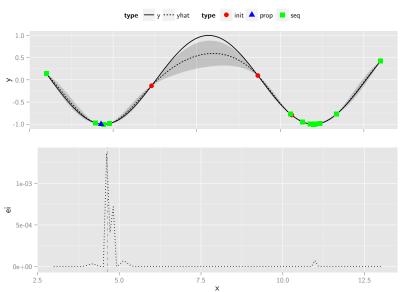
Iter = 3, Gap = 5.5410e-02



Iter = 4, Gap = 2.2202e-05

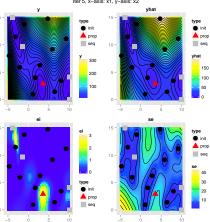


Iter = 5, Gap = 2.2202e-05



MLRMBO: MODEL-BASED OPTIMIZATION TOOLBOX

- ► Any regression from mlr
- ► Arbtritrary infill
- ► Single or multi-crit
- Multi-point proposal
- Via parallelMap and batchtools runs on many parallel backends and clusters
- Algorithm configuration
- Active research

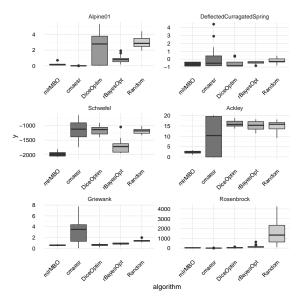


- mlr: https://github.com/mlr-org/mlr
- mlrMBO: https://github.com/mlr-org/mlrMBO
- mlrMBO Paper on arXiv (under review) https://arxiv.org/abs/1703.03373

BENCHMARK MBO ON ARTIFICIAL TEST FUNCTIONS

- ► Comparison of mIrMBO on multiple different test functions
 - Multimodal
 - Smooth
 - Fully numeric
 - Well known
- ► We use GPs with
 - LCB with $\lambda = 1$
 - Focussearch
 - 200 iterations
 - ▶ 25 point initial design, created by LHS sampling
- ► Comparison with
 - Random search
 - CMAES
 - ► other MBO implementations in R

MBO GP VS. COMPETITORS IN 5D



${\sf Section}\ 2$

PARALLEL BATCH PROPOSALS

MOTIVATION FOR BATCH PROPOSAL

- Function evaluations expensive
- Often many cores available on a cluster
- ► Underlying *f* can in many cases not be easily parallelized
- ► Natural to consider batch proposal
- ▶ Parallel MBO: suggest q promising points to evaluate: x_1^*, \ldots, x_q^*
- \blacktriangleright We need to balance exploration and exploitation
- ► Non-trivial to construct infill criterion for this

REVIEW OF PARALLEL MBO STRATEGIES

- ► Constant liar: (Ginsbourger et al., 2010)
 - ► Fit kriging model based on real data and find x₁^{*} according to El-criterion.
 - "Guess" $f(x_{i-1}^*)$, update the model and find x_i^* , i = 2, ..., q
 - ► Use f_{min} for "guessing"
- ▶ *q*-**LCB**: (Hutter et al., 2012)
 - q times: sample λ from Exp(1) and optimize single LCB criterion
 - ► $\mathbf{x}^* = \operatorname{arg\,min}_{\mathbf{x} \in \mathcal{X}} \operatorname{LCB}(\mathbf{x}) = \operatorname{arg\,min}_{\mathbf{x} \in \mathcal{X}} \hat{f}(\mathbf{x}) \lambda \hat{s}(\mathbf{x})$.

Multiobjectivization

- Originates from multi-modal optimization
- ► Add distance to neighbors for current set as artificial objective
- Use multiobjective optimization
- ► Select by hypervolume or first objective or ...

► Our approach

- ► Decouple f̂(x) and ŝ(x) as objectives instead of EI to have different exploration / exploitation trade-offs
- ► Consider distance measure as potential extra objective
- Run multiobjective EA to select q well-performing, diverse points
- Distance is possible alternative if no or bad $\hat{s}(x)$ estimator
- ► Decoupling y(x), ŝ(x) potential alternative when El derivation does not hold for other model classes

Bischl, Wessing et al: *MOI-MBO: Multiobjective infill for parallel model-based optimization*, LION 2014

EXPERIMENTAL SETUP

Problem Instances

- All 24 test functions of the black-box optimization benchmark (BBOB) noise-free test suite
- Dimensions $d \in \{5, 10\}$

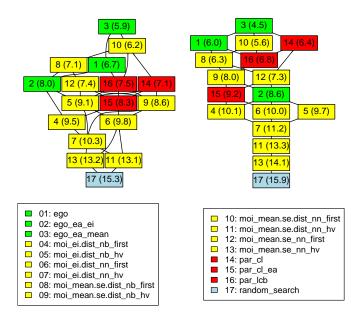
Budget

- ► For every function 10 initial designs of size 5 · d ⇒ 10 statistical replications for each problem instance
- ▶ $40 \cdot d$ function evaluations on top of the initial design
- Parallel optimization: batches of size q = 5

Visualization: Preference relation graph

- Each node represents an approach (mean rank in braces)
- Two nodes are connected with an edge if one approach (the upper) is significantly better than the other (the lower) according to the sign test

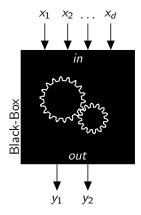
Result Graphs



Section 3

MULTICRITERIA SMBO

MODEL-BASED MULTI-OBJECTIVE OPTIMIZATION



 $\min_{\boldsymbol{x} \in \mathbb{X}} \mathbf{f}(\boldsymbol{x}) = \boldsymbol{y} = (y_1, ..., y_m) \text{ with } \mathbf{f} : \mathbb{R}^d \to \mathbb{R}^m$ $\blacktriangleright \boldsymbol{y} \text{ dominates } \tilde{\boldsymbol{y}} \text{ if } (3)$

$$\forall i \in \{1, ..., m\} : y_i \leq \tilde{y}_i \qquad (4)$$

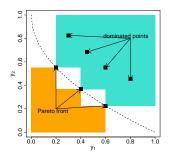
and
$$\exists i \in \{1, ..., m\} : y_i < \tilde{y}_i$$
 (5)

Set of non-dominated solutions:

 $\mathcal{X}^* := \{ x \in \mathcal{X} | \nexists \tilde{x} \in \mathcal{X} : f(\tilde{x}) \text{ dominates } f(x) \}$

- ▶ Pareto set \mathcal{X}^* , Pareto front $\mathbf{f}(\mathcal{X}^*)$
- ▶ Goal: Find X̂* of non-dominated points that estimates the true set X*

MODEL-BASED MULTI-OBJECTIVE OPTIMIZATION



$$\min_{\mathbf{x} \in \mathbb{X}} \mathbf{f}(\mathbf{x}) = \mathbf{y} = (y_1, ..., y_m) \text{ with } \mathbf{f} : \mathbb{R}^n \to \mathbb{R}^m$$

$$\blacktriangleright \mathbf{y} \text{ dominates } \tilde{\mathbf{y}} \text{ if } \tag{6}$$

$$\forall i \in \{1, ..., m\} : y_i \leq \tilde{y}_i \qquad (7)$$

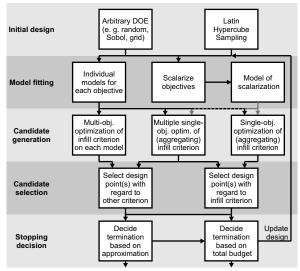
and
$$\exists i \in \{1, ..., m\} : y_i < \tilde{y}_i$$
 (8)

Set of non-dominated solutions:

 $\mathcal{X}^* := \{ \textbf{x} \in \mathcal{X} | \nexists \tilde{\textbf{x}} \in \mathcal{X} : \textbf{f}(\tilde{\textbf{x}}) \text{ dominates } \textbf{f}(\textbf{x}) \}$

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TAXONOMY



Horn, Wagner, Bischl et al: *Model-based multi-objective optimization: Taxonomy, multi-point proposal, toolbox and benchmark*, EMO 2014

BATCH PROPOSAL

- Most MBMO lack way to propose N > 1 points (batch evaluation)
- Batch evaluations are essential for distributed computing
- ▶ We integrated such mechanism(s) for arbitrary MBMO
- Replaced single phases of the taxonomy

PAREGO

1. Scalarize objectives using the augmented Tchebycheff norm

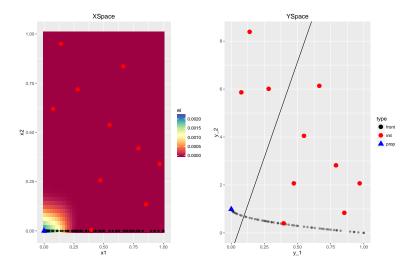
$$\max_{i=1,...,d} [w_i f_i(\mathbf{x})] + \rho \sum_{i=1}^d w_i f_i(\mathbf{x})$$

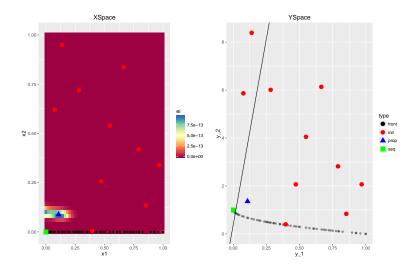
with uniformly distributed weight vector $\mathbf{w} (\sum w_i = 1)$ and fit surrogate model to the respective scalarization.

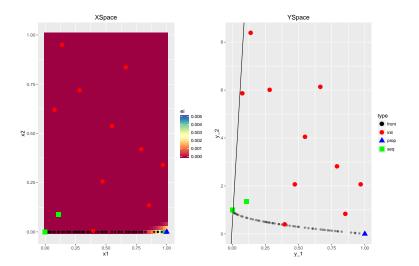
2. Single-objective optimization of EI (or LCB?)

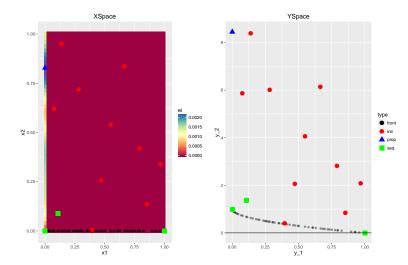
Batch proposal: Increase the number and diversity of randomly drawn weight vectors

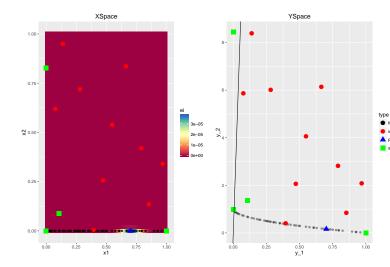
- If N points are desired, cN (c > 1) weight vectors are considered
- Greedily reduce set of weight vectors by excluding one vector of the pair with minimum distance
- ► Scalarizations implied by each weight vector are computed
- ► Fit and optimize models for each scalarization
- Optima of each model build the batch to be evaluated









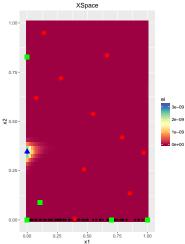


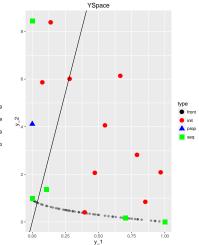
front

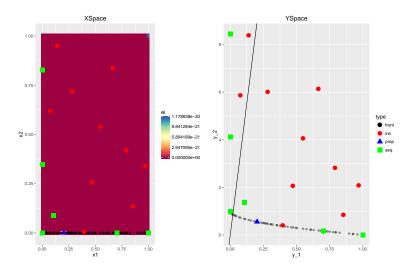
init

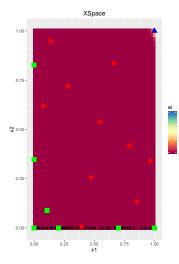
prop

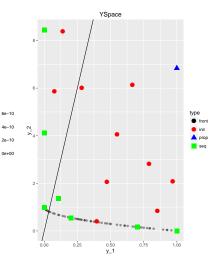
seq

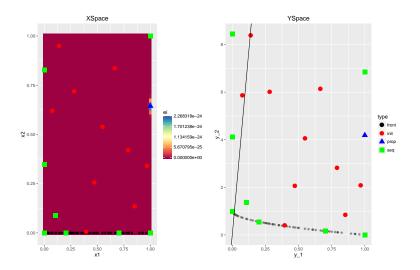


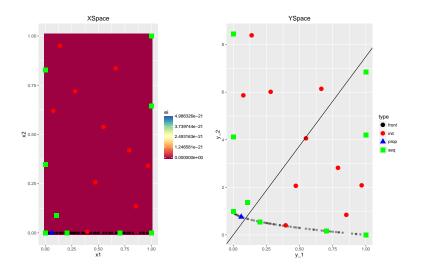












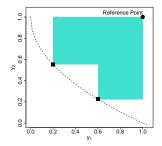
SMS-EGO

- Individual models for each objective
- Single-objective optimization of aggregating infill criterion: Calculate contribution of the confidence bound of representative solution to the current front approximation
- ► Calculate LCB for each objective
- Measure contribution with regard to the hypervolume indicator
- For ε-dominated (≤ε) solutions, a penalty

$$\Psi(\mathbf{x}) = -1 + \prod_{j=1}^{m} \left(1 + (I(\mathbf{x}) - y_j^{(i)}) \right)$$

is added

(Actually not needed for Focussearch.)



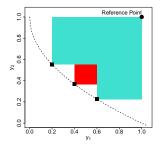
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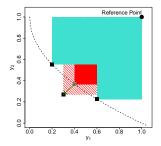


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 is added

(Actually not needed for Focussearch.)



Modification of phase candidate generation: Use simulated evaluations for candidate generation

- ► The proposed point x^{*} is not directly evaluated, but the LCB I(x^{*}) is added to the current approximation without refitting the model
- Repeat until N points for a batch evaluation have been found

APPROXIMATIVE RBF-SVM TRAINING ALGORITHMS

-

SVM solver	Description
Pegasos	Stochastic Gradient Descent
BSGD	Budgeted Stochastic Gradient Descent
LLSVM	Low-rank kernel approximation + linear solver
LIBSVM	"Exact" SMO solver
LASVM	Online variant of SMO solver
LIBBVM/CVM	Minimum Enclosing Ball (only squared hinge loss)
SVMperf	Cutting Plane Algorithm

- ▶ What is the trade-off between training time and prediction error?
- \blacktriangleright Most solvers have 2 additional parameters on top of C and γ
- ► Optimizing 2 expensive objectives in a 4-dim parameter space.
- ► Replace grid search with more sophisticated PAREGO-algorithm.

APPROXIMATIVE SVM TRAINING ALGORITHMS

- ► We expect: Every solver has a trade-off between training time and prediction error: Given more time a solver (should) reach a better solution.
- Our goal: Analyze this trade-off! Solve the multi-criteria optimization problem with respect to the two objectives error and training time by varying the parameters.
- ► **The challenge:** Optimizing 2 expensive objectives in a 4-dimensional parameter space.
- Our approach: Replace standard grid search with more sophisticated PAREGO-algorithm.

APPROXIMATIVE SVM TRAINING ALGORITHMS

The parameters (C, γ) of the SVM itself were optimized over $2^{[-15,15]}$ respectively. Every solver has further approximation parameters:

SVM solver	Parameters	Optimization Space	
Pegasos	#Epochs	2 ^[0,7]	
BGSD	Budget size, #Epochs	2 ^[4,11] , 2 ^[0,7]	
LLSVM	Matrix rank	2 ^[4,11]	
LIBSVM	ϵ (Accuracy)	$2^{[-13,-1]}$	
LASVM	ϵ (Accuracy), #Epochs	$2^{[-13,-1]}, 2^{[0,7]}$	
LIBBVM/CVM	ϵ (Accuracy)	$2^{[-19,-1]}$	
SVMperf	ϵ (Accuracy), #Cutting planes	$2^{[-13,-1]}$, $2^{[4,11]}$	

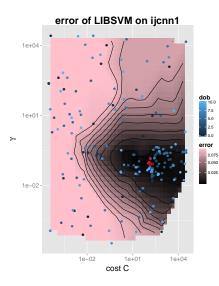
Additional parameters set to default values.

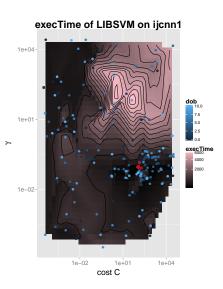
DATASETS

data set	# points	# features	class ratio	sparsity
wXa	34 780	300	34.45	95.19 %
aXa	36 974	123	3.17	88.72 %
protein	42 153	357	1.16	71.46 %
mnist	70 000	780	0.96	80.76 %
vehicle	98 528	100	1.00	0 %
shuttle	101 500	9	0.27	0.23 %
spektren	175 090	22	0.80	0 %
ijcnn1	176691	22	9.41	40.91 %
arthrosis	262 142	178	1.19	0.01 %
cod-rna	488 565	8	2.00	0.02 %
covtype	581012	54	1.05	78 %
poker	1025010	10	1.00	0 %

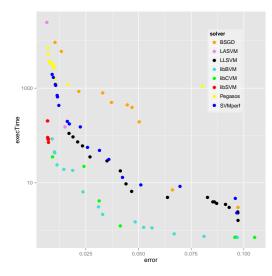
 TABLE : Overview of the data sets.

TEST ERROR LANDSCAPE (LIBSVM)





ALL PARETO FRONTS FOR IJCNN1 DATASET



- 176 691 samples
 22 features
 9.41 class ratio
- Wee see: LIBSVM: exact but slow LIBBVM/CVM: good front - speed increase with small accuracy loss SVMperf / LLSVM / BSGD: can be really fast, but higher accuracy loss LASVM / Pegasos: less exact and even slower as LIBSVM

Section 4

INTERESTING CHALLENGES

CHALLENGE: THE CORRECT SURROGATE?

- ► GPs are very much tailored to what we want to do, due to their spatial structure in the kernel and the uncertainty estimator.
- But GPs are rather slow. And (fortunately) due to parallization (or speed-up tricks like subsampling) we have more design points to train on.
- Categorical features are also a problem in GPs (although methods exist, usually by changing the kernel)
- Random Forests handle categorical features nicely, are much faster. But they don't rely on a spatial kernel and the uncertainty estimation is much more heuristic / may not represent what we want.

CHALLENGE: TIME HETEROGENEITY

- Complex configuration spaces across many algorithms results in vastly different runtimes in design points.
- Actually just the RBF-SVM tuning can result in very different runtimes.
- ► We don't care how many points we evaluate, we care about total walltime of the configuration.
- The option to subsample further complicates things.
- ► Parallelization further complicates things.
- Option: Estimate runtime as well with a surrogate, integrate it into acquisition function.

Section 5

ML MODEL SELECTION AND HYPERPARAMETER OPTIMIZATION

Automatic Model Selection

PRIOR APPROACHES:

- ► Looking for the silver bullet model → Failure
- Exhaustive benchmarking / search
 ~ Very expensive, often contradicting results
- Meta-Learning:
 - \rightsquigarrow Good meta-features are hard to construct
 - \rightsquigarrow IMHO: Gets more interesting when combined with SMBO

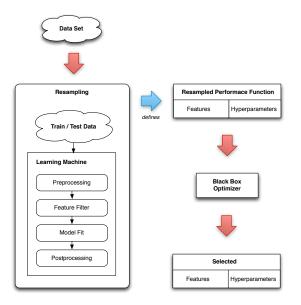
GOAL FOR AUTOML:

- ► Data dependent
- ► Automatic
- ► Include every relevant modeling decision
- ► Efficient
- ► Learn on the model-settings level!

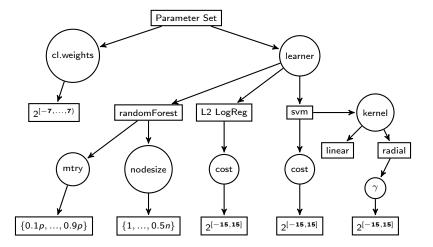
FROM NORMAL SMBO TO HYPERARAMETER TUNING

- ► Objective function is resampled performance measure
- ► Parameter space θ ∈ Θ might be discrete and dependent / hierarchical
- No derivative for $f(\cdot, \theta)$, black-box
- Objective is stochastic / noisy
- Objective is expensive to evaluate
- ► In general we face a problem of algorithm configuration:
- \blacktriangleright \rightsquigarrow Usual approaches: racing or model-based / bayesian optimization

FROM NORMAL SMBO TO HYPERARAMETER TUNING



COMPLEX PARAMETER SPACE



FROM NORMAL SMBO TO HYPERARAMETER TUNING

- ► Initial design: LHS principle can be extended, or just use random
- Focus search: Can be (easily) extended, as it is based on random search. To zoom in for categorical parameters we randomly drop a category for each param which is not present in the currently best configuration.
- ► Few approaches for GPs with categorical params exist (usually with new covar kernels), not very established
- ► Alternative: Random regression forest (mlrMBO, SMAC)
- Estimate uncertainty / confidence interval for mean response by efficient bootstrap technique¹, or jackknife, so we can define EI(x) for the RF
- Dependent params in mlrMBO: Imputation:
- Many of the current techniques to handle these problems are (from a theoretical standpoint) somewhat crude

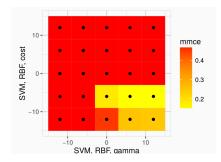
¹Sexton et al, "Standard errors for bagged and random forest estimators, 2009."

HYPERPARAMETER TUNING

► Still common practice: grid seach For a SVM it might look like:

- $\begin{array}{l} \blacktriangleright \quad C \in (2^{-12}, 2^{-10}, 2^{-8}, \dots, 2^8, 2^{10}, 2^{12}) \\ \blacktriangleright \quad \gamma \in (2^{-12}, 2^{-10}, 2^{-8}, \dots, 2^8, 2^{10}, 2^{12}) \end{array}$
- Evaluate all $13^2 = 169$ combinations $C \times \gamma$
- Bad beacause:
 - optimum might be "off the grid"
 - lots of evaluations in bad areas
 - lots of costy evaluations
- ► How bad?

Hyperparameter Tuning

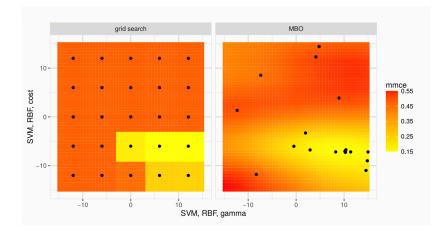


- ▶ Because of budget restrictions grid might even be smaller!
- Unpromising area quite big!
- Lots of costly evaluations!

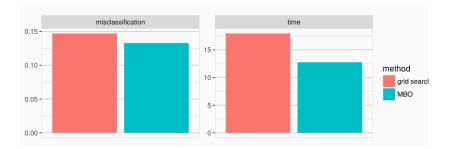
With mlrMBO it is not hard to do it better! More interesting applications to time-series regression and cost-sensitive classification^2 $\,$

²Koch, Bischl et al: *Tuning and evolution of support vector kernels*, EI 2012

Hyperparameter Tuning



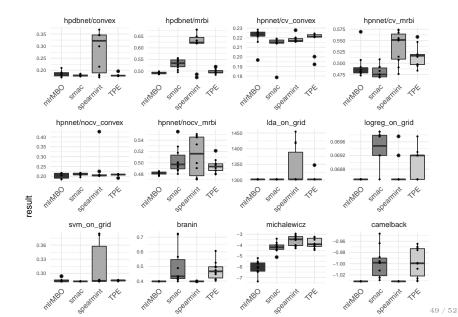
Hyperparameter Tuning



HPOLIB

- HPOlib is a set of standard benchmarks for hyperparameter optimizer
- Allows comparison with
 - Spearmint
 - SMAC
 - Hyperopt (TPE)
- ► Benchmarks:
 - ► Numeric test functions (similar to the ones we've seen bevor)
 - ► Numeric machine learning problems (Ida, SVM, logistic regression)
 - Deep neural networks and deep belief networks with 15 and 35 parameters.
- For benchmarks with discrete and dependent parameters (hpnnet, hpdbnet) a random forest with standard error estimation is used.

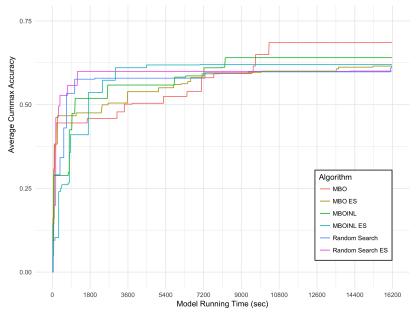
MBO: HPOLIB



DEEP LEARNING CONFIGURATION EXAMPLE

- Dataset: CIFAR-10 (60000 32x32 images with 3 color channels; 10 classes)
- Configuration of a deep neural network (mxnet)
- Size of parameter set: 30, including number of hidden layers, activation functions, regularization, convolution layer setting, etc.
- ▶ Split: 2/3 training set, 1/6 test set, 1/6 validation set
- ► Time budget per tuning run: 4.5h (16200 sec)
- ► Surrogate: Random forest
- Acquisition: LCB with $\lambda = 2$

DEEP LEARNING CONFIGURATION EXAMPLE



Thanks! Any comments or questions?