Bayesian semiparametric vector autoregressive models

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(joint work with Professor Jim Griffin)

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Outline

1. Vector autoregressive models (VARs)
2. Motivation for nonlinear VARs
3. Bayesian nonparametric methods
4. The Bayesian semiparametric VAR(1)
5. Computation
6. Empirical examples
7. Conclusion/Discussion

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popularity - 'reduced form' VAR

- Introduced by Sims (1980), VAR is used by macroeconomists
  - to characterise the joint dynamic behaviour of a collection of variables, and
  - to forecast movements of macroeconomic variables based on potential future paths of specified variables.
- We focus on the 'reduced' form VAR, i.e. the stationary VAR model without restrictions,

$$\boldsymbol{y}_t = \boldsymbol{B}\boldsymbol{y}_{t-1} + \epsilon_t \tag{1}$$

for $t = 1, \ldots, T$, where

$\boldsymbol{y}_t = (y_{1,t}, y_{2,t}, \ldots, y_{m,t})'$ is the $m \times 1$ vector of macroeconomic variables at time t,

$\boldsymbol{B}$ is the $m \times m$ matrix of unknown regression coefficients,

$\epsilon_t = (\epsilon_{1,t}, \epsilon_{2,t}, \ldots, \epsilon_{m,t})'$ is the $m \times 1$ innovation vector at time t.

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- Past and current literature finds departures from the linear and Gaussian VAR form. See Koop et al. (1996), Weise(1999), Favero (2012), Wong (2013) to name a few.
- The main argument is that linear models cannot adequately capture 'asymmetries' that may exist in business cycle fluctuations.
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- We want to merge these two sides of literature, and consider non-linear, semi-parametric VAR models.
- The model we propose uses Bayesian non-parametric methods.
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- A nonparametric model has an infinite number of parameters. For example, in density estimation the parameters can consist of all densities.
- In Bayesian statistics, we need to define a prior for the parameters. This is non trivial since we are working on an infinite parameter space.
- The solution, is to define a stochastic process to be your prior. The Dirichlet process (DP) introduced in Ferguson (1973) is the popular Bayesian nonparametric prior, and the one we will use for our VAR model.

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Dirichlet process mixture model (DPM)

- We the DPM to construct non-linear first order stationary multivariate VAR processes with non-Gaussian innovations. So what is the DPM?
- Introduced by Lo (1984) the DPM model, with Gaussian kernel, is given by

$$f_P(y) = \int \mathrm{N}(y; \mu, \sigma^2)\, \mathrm{d}P(\phi)$$

where $P \sim D(M, P_0)$ - a DP with precision parameter $M > 0$, and base measure, P_0, a distribution on $\mathbb{R} \times \mathbb{R}_+$,

and $\phi = (\mu, \sigma^2)$ with μ to represent the mean and σ^2 the variance of the normal component.

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- The DPM's stick-breaking representation (see Sethuraman (1992)), is :

$$P = \sum_{j=1}^{\infty} w_j \, \delta_{\phi_j},$$

$\phi_j \overset{\text{iid}}{\sim} P_0$, $w_1 = v_1$, $w_j = v_j \prod_{l<j}(1 - v_l)$ with $v_j \overset{\text{iid}}{\sim} \text{Be}(1, M)$, and we can write

$$f_{v,\phi}(y_i) = \sum_{j=1}^{\infty} w_j \, \text{N}(y_i; \phi_j)$$

To facilitate computation auxiliary allocation variables $(d_1, \ldots, d_n)$ are introduced, so that $p(d_i = j) = w_j$, and $w_1, w_2, \ldots \perp\!\!\!\perp \phi_1, \phi_2 \ldots$

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Model construction

- Villalobos and Walker (2013), construct a Bayesian nonparametric version of the AR(1) model, by expressing both the joint and transition densities as DPM.
- We extend their idea to the multivariate case and model the transition and joint densities using the DPM.
- The joint density will therefore be:

$$f\begin{pmatrix} y_{t-1} \\ y_t \end{pmatrix} = \sum_{j=1}^{\infty} w_j \, \mathrm{N}\left(\begin{pmatrix} y_{t-1} \\ y_t \end{pmatrix} \middle| \begin{pmatrix} \mu_j \\ \mu_j \end{pmatrix} \begin{pmatrix} \Sigma_j & \Omega_j \\ \Omega_j & \Sigma_j \end{pmatrix} \right)$$

  for $j = 1, 2, \ldots, t = 1, \ldots, T$, and $i = 1, \ldots, m$.

- The stationary distribution is then $f(y_t) = \sum_{j=1}^{\infty} \mathrm{N}(y_t | \mu_j, \Sigma_j)$

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- The stationary distribution is then $f(y_t) = \sum_{j=1}^{\infty} \mathrm{N}(y_t | \mu_j, \Sigma_j)$

- The y_t and y_{t-1} are m dimensional vectors,
- the μ_j's are also m dimensional vectors, and we have an infinite number of them,
- the Σ_j's are $m \times m$ positive definite matrices, and we have an infinite number of them,
- and the Ω_j's are $m \times m$ matrices, and we also have an infinite number of them.

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- The transition density will then be

$$f(y_t|y_{t-1}) = \frac{\sum_{j=1}^{\infty} w_j \, \mathrm{N}\left(\left(\begin{array}{c} y_{t-1} \\ y_t \end{array} \right) \middle| \left(\begin{array}{c} \mu_j \\ \mu_j \end{array} \right) \left(\begin{array}{cc} \Sigma_j & \Omega_j \\ \Omega_j & \Sigma_j \end{array} \right) \right)}{\sum_{j=1}^{\infty} w_j \, \mathrm{N}(y_{t-1}|\mu_j, \Sigma_j)}$$

- We can then re-write it as locally weighted mixture of VAR(1)'s as follows,

$$f(y_t|y_{t-1}) = \sum_{j=1}^{\infty} p_j(y_{t-1}) \, \mathrm{N}(y_t|\theta_j(y_{t-1}), \Lambda_j(y_{t-1}))$$

where,

- $p_j(y_{t-1}) = \dfrac{w_j \, \mathrm{N}(y_{t-1} | \mu_j, \Sigma_j)}{\sum_{k=1}^{\infty} \mathrm{N}(y_{t-1} | \mu_k, \Sigma_k)}$,
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Priors

- When selecting the priors, we must ensure that the Σ_j matrices are positive definite. For this reason rather than placing priors on the variance covariance matrices, we decompose them in terms of correlation matrices.
- We follow Karlsson (2012), and write Σ_j, Ω_j as follows:

  $\Sigma_j = S_j P_j S_j$ and $\Omega_j = S_j R_j S_j$

  - where, S_j is an $m \times m$ diagonal matrix , with $\sigma_{1,j}, \ldots, \sigma_{m,j}$, in the diagonal.
  - P_j is the $m \times m$ correlation matrix at time t of the VAR(1) variables. Its elements are the correlations between $y_{t,k}$ and $y_{t,l}$ in the j^{th} component.
  - R_j is the cross-correlation matrix at times t and $t-1$. Its elements are the correlations between $y_{t-1,k}$ and $y_{t,l}$ in the j^{th} component.

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key points

- Standard MCMC methods for infinite mixture models cannot be used here.
- We use the adaptive truncation method of Griffin(2013).
- We sample a sequence of posteriors for truncated versions of the model, with different levels of truncation.
- The algorithm provides a method for choosing when to stop sampling this sequence, in such a way so that large truncation errors are avoided. The final posterior provides an approximation to the posterior of the infinite dimensional model.

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data

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- The sample period is from the second quarter of 1965 to the first quarter of 2011.
- The three variables are:
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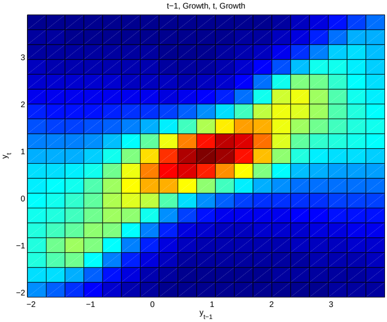
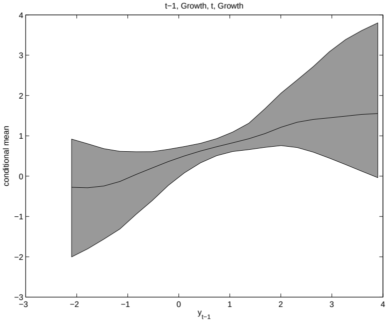
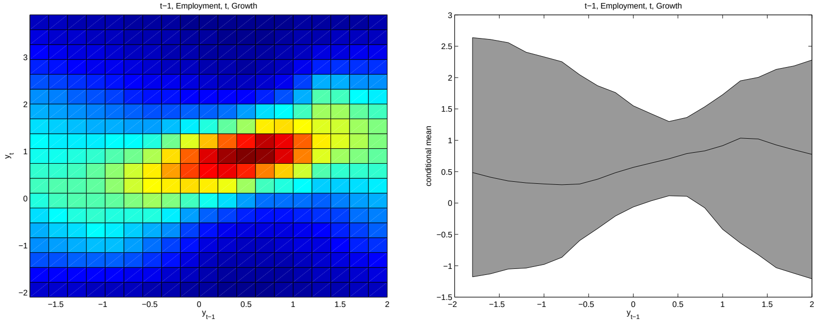
GDP growth at t and $t-1$

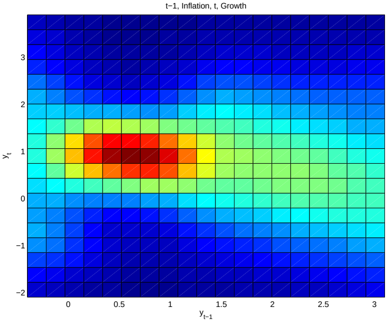


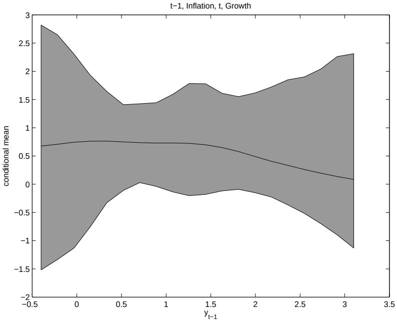


GDP growth at t and Employment growth at $t-1$

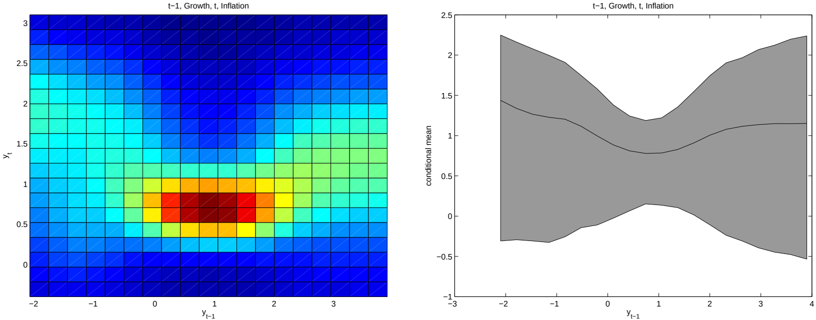


GDP growth at t and GDP deflator at $t-1$

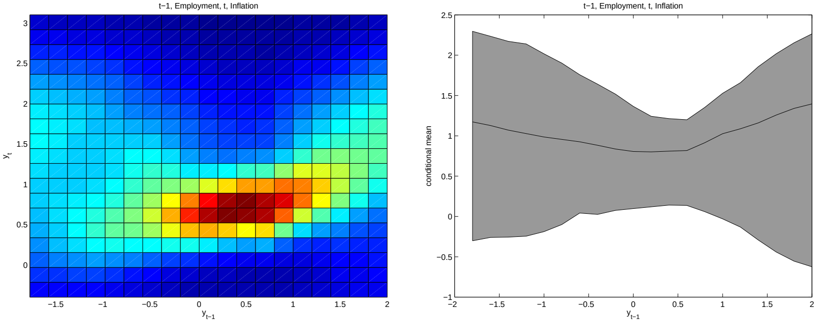




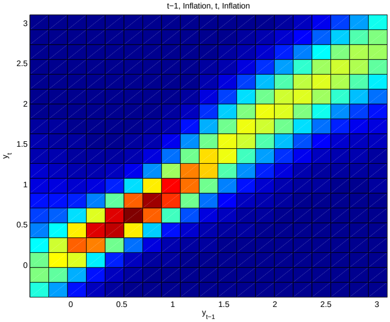
GDP deflator at t and GDP growth at $t-1$

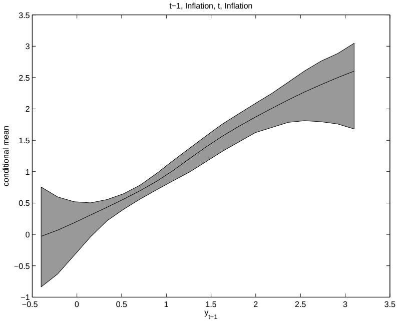


GDP deflator at t and Employment growth at $t-1$

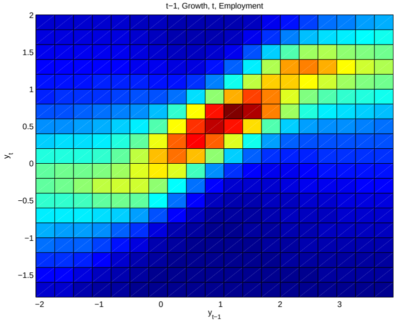


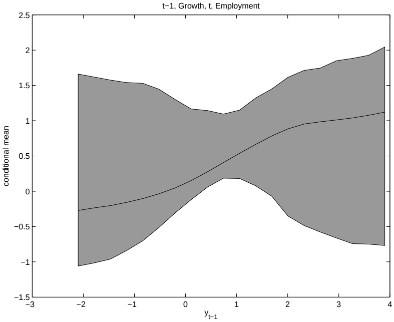
GDP deflator at t and $t-1$



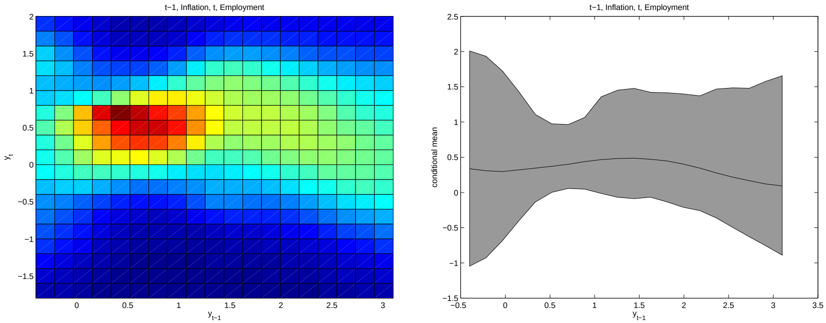


Employment growth at t and GDP growth at $t-1$





Employment growth at t and GDP deflator at $t-1$



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