Mutual Fund Performance When Investors Learn About Skill

Mark Fisher[†], Mark J. Jensen[†] and Paula Tkac[†]

[†]Federal Reserve Bank of Atlanta
March 20, 2015

The views expressed here are mine and not necessarily those of the Federal Reserve Bank of Atlanta nor the Federal Reserve System.

I. Past Empirical Findings on Mutual Fund Performance

Assuming each fund's alpha are not draws from an underlying population distribution, Jensen (1968); Lehmann and Modest (1987); Elton et al. (1993); Gruber (1996); Elton, Gruber and Blake (1996); Carhart (1997); Pastor and Stambaugh (2002a, 2002b); Elton, Gruber and Blake (2003); and Elton, Gruber and Blake (2011) find that net of fees active funds on average under perform a passive fund by 65 to 200 basis points per year.

Using fixed population distributions for skill, Baks, Metrick and Wachter (2001) conclude that investing in actively managed funds cannot be justified solely on the statistical evidence, but requires extremely skeptical beliefs about funds being unskilled.

Modeling fund's returns with increasing costs, a normal, population, distribution of skill, Berk and Green (2004) find the average population level of skill just covers fees and transaction costs. In other words, active funds collect the rents from their extraordinary skill.

II. Past Empirical Findings on Mutual Fund Performance

Jones and Shanken (2005) assume mutual fund skill is normally distributed across the universe of funds but with unknown mean and variance. Investors learn across funds about the population mean and variance of skill. Funds are found to possess some skill but not enough to offset their fees and expenses, but these findings are conditional on a non-informative prior for the population mean and variance.

Kosowski, Timmerman, Wermers and White (2006), Barras, Scaillet and Wermers (2010) and Fama and French (2010) bootstrap the empirical cross-sectional distribution of skill from the OLS estimates of each fund's alpha and find a few significantly skilled and unskilled mutual funds.

- How skill is distributed over the population of actively managed mutual funds, and how one estimates this distribution, impacts whether actively managed funds are skilled or not.

Contribution

Our contribution is modelling the population distribution of mutual fund skill as an unknown measure and endowing investors with a prior for this unknown distribution that assumes very little about the unknown distribution of skill.

The cross-sectional distribution of skill contains information about a "typical" fund's potential level of skill.

Return histories from extraordinary skilled and unskilled funds add information to the cross-sectional distribution about the extraordinary funds potential skill.

There are fewer highly skilled and unskilled funds than the, "are they skilled or are they just lucky?" literature thought.

There are more highly skilled and unskilled funds than a normal, cross-sectional, distribution of skill suggests.

Risk-Factor Model of Mutual Fund Returns

Excess monthly gross returns for the ith mutual fund follows

$$r_{i,t} = \alpha_i + \beta_i' F_t + \sigma_i \epsilon_{i,t}$$

where $i = 1, \ldots, J$ and $t = 1, \ldots, T_i$.

- α_i is the skill level of the mutual fund manager
- F_t are the common risk-factors, consisting of the three Fama-French risk-factors of the market (MRK), size (SMB), and value (HML) portfolios, and Carhart's momentum (MOM) portfolio.
- $\epsilon_{i,t} \overset{iid}{\sim} N(0,1)$ and $\epsilon_{i,t} \perp \epsilon_{i',t}$; i.e., no fund conveys information about any other fund.

I. Population Distribution of Skill

Investors prior belief for the population distribution of mutual fund skill, alpha, is

$$\pi(\alpha) = \int N(\alpha|\mu_\alpha, \sigma_\alpha^2) \, dG(\mu_\alpha, \sigma_\alpha^2).$$

Which has the hierarchical representation

$$\begin{aligned} \alpha|\mu_\alpha, \sigma_\alpha^2 &\sim N(\mu_\alpha, \sigma_\alpha^2), \\ \mu_\alpha, \sigma_\alpha^2 &\sim G. \end{aligned}$$

What to assume about G? For example, see Gelman (2006) and Frühwirth-Schnatter and Wagner (2010).

II. Bek et al., 2001 Population Distribution

- **No learning about $\mu_\alpha, \sigma_\alpha$:** Baks et al., 2001 (BMW) and Pastor & Stambaugh, 2002 assume

$$G(\mu_\alpha, \sigma_\alpha^2) = \delta_{m_0, s_0^2}$$

for a fixed population mean m_0 and variance s_0^2. Thus, the prior and posterior population distribution of skill is

$$\pi(\alpha) = N(m_0, s_0^2).$$

Investors who do not believe the α_is come from a common population distribution would set $m_0 = 0$ and let $s_0^2 \to \infty$; i.e., let $\pi(\alpha)$ be a uniform, cross-sectional, distribution over the real line so that each fund's skill is independent from the other funds.

I. Jones & Shanken's Population Distribution of Skill

- **Learning about $\mu_\alpha, \sigma_\alpha$:** Jones and Shanken, 2005 assume

$$G(\mu_\alpha, \sigma_\alpha^2) = \text{Normal-Inv-Gamma}\left(m_0, \frac{\sigma_\alpha^2}{\kappa_0}, \frac{\nu_0}{2}, \frac{\nu_0}{2} s_0^2\right).$$

The investor's initial population distribution for alpha is

$$\pi(\alpha) = t_{\nu_0}\left(\alpha \middle| m_0, \left(\frac{\kappa_0 + 1}{\kappa_0}\right) \nu_0 s_0^2\right).$$

The posterior population distribution, $\pi(\alpha|\alpha_1, \ldots, \alpha_J)$ will be a Student-t with $\nu_0 + J$ degrees of freedom, mean $\frac{\kappa_0 m_0 + J\overline{\alpha}}{\kappa_0 + J}$ and scale,

$$\frac{\kappa_0 + J + 1}{\kappa_0 + J}\left\{\frac{\nu_0 s_0^2 + \sum_j (\alpha_j - \overline{\alpha})^2 + \frac{J\kappa_0}{\kappa_0 + J}(m_0 - \overline{\alpha})^2}{\nu_0 + J}\right\}$$

II. Jones & Shanken's Population Distribution of Skill

- So investors posterior understanding for the cross-sectional distribution of mutual fund skill depends on the investors choice for the hyperparameters $m_0, \kappa_0, \nu_0, s_0^2$, and on actual mutual fund performance, $\alpha_1, \ldots, \alpha_J$.
- The posterior distribution of the jth mutual fund level alpha is independent of the other funds, but it will contain information found in the performance of the other funds through the investors posterior for the population mean and variance, $\pi(\alpha_i) = \int N(\alpha_i|\mu_\alpha, \sigma_\alpha^2) d\pi(\mu_\alpha, \sigma_\alpha^2|\alpha_{-i})$.
- Investors believe that the performance of all funds come from a normal distribution with the same population mean and variance.
- Uninformed investors have a prior for the population mean and variance proportional to $\pi(\mu_\alpha, \sigma_\alpha^2) \propto 1/\sigma_\alpha^2$. But even uninformed investors continue to believe skill is normally distributed over the entire universe of mutual funds.

Learning the Population Distribution of Skill

Learning about $G(\mu_\alpha, \sigma^2_\alpha)$: Our investors are completely ignorant about G but have a prior

$$\alpha|\mu_\alpha, \sigma^2_\alpha \sim N(\mu_\alpha, \sigma_\alpha), \tag{1}$$

$$\mu_\alpha, \sigma^2_\alpha \sim G, \tag{2}$$

$$G|B, G_0 \sim DP(B, G_0), \tag{3}$$

where DP denotes a Dirichlet Process distribution with G_0 being the investors initial guess for G; i.e., $E[G] = G_0$. B is a non-negative scalar for how confident the investor is in the guess of G_0 since

$$\text{Var}[G] = \frac{G_0(1 - G_0)}{1 + B}.$$

Initial Guess For the Distribution of Alphas

Since $E[G] = G_0$ the initial guess for the population distribution of alpha is

$$\widehat{\pi}(\alpha) \equiv E[\pi(\alpha)] \quad = \quad \int N(\alpha|\mu_\alpha, \sigma_\alpha^2) dG_0(\mu_\alpha, \sigma_\alpha^2)$$

If

$$G_0 \equiv \text{Normal-Inverse-Gamma}(m_0, \sigma_\alpha^2/\kappa_0, \nu_0/2, \nu_0 s_0^2/2)$$

then

$$\widehat{\pi}(\alpha) = t_{\nu_0}\left(\alpha \,\middle|\, m_0, \left(\frac{\kappa_0+1}{\kappa_0}\right)\nu_0 s_0^2\right).$$

JS and our investor have the same initial t-distribution for the cross-sectional distribution of skill, so why use the DP prior?

I. Updating the Distribution of Alphas

The investor observes a draw of (μ_1, σ_1) from G for the first mutual fund. Given (μ_1, σ_1), investors update their beliefs for G to

$$G|\mu_1, \sigma_1^2 \sim DP(B+1, G_0 + \delta_{\mu_1, \sigma_1^2}).$$

The investors new guess for G is $G_0 + \delta_{\mu_1,\sigma_1^2}$ and for the population distribution is

$$\begin{aligned}
\left[\widehat{\pi}(\alpha)|\mu_1, \sigma_1^2\right] &= \int N(\alpha|\mu_\alpha, \sigma_\alpha^2) dE\left[G(\mu_\alpha, \sigma_\alpha^2)|\mu_1, \sigma_1^2\right] \\
&= \frac{B}{1+B} t_{\nu_0}\left(\alpha \,\middle|\, m_0, \left(\frac{\kappa_0+1}{\kappa_0}\right)\nu_0 s_0^2\right) \\
&\quad + \frac{1}{1+B} N(\alpha|\mu_1, \sigma_1^2)
\end{aligned}$$

a mixure of the prior population distribution for alpha and the empirical normal distribution with the first mutual funds expected skill and variance.

- $[\widehat{\pi}(\alpha)|\mu_1, \sigma_1^2]$ is the best guess at the potential skill of the next mutual fund.

II. Updating the Distribution of Alphas

After observing (μ_i, σ_i^2), $i = 1, \dots, J$, the posterior for G is

$$G|\mu_1, \sigma_1^2, \dots, \mu_J, \sigma_J^2 \sim DP\left(B + J, G_0 + \sum_{i=1}^{J} \delta_{\mu_i, \sigma_i^2}\right)$$

and investors posterior guess for how skill is distributed across the universe of mutual funds is

$$\begin{aligned}[\widehat{\pi}(\alpha)|\mu_1, \sigma_1^2, \dots, \mu_J, \sigma_J^2] &= \frac{B}{J+B} t_{\nu_0}\left(\alpha \middle| m_0, \left(\frac{\kappa_0 + 1}{\kappa_0}\right) \nu_0 s_0^2\right) \\ &\quad + \frac{1}{J+B} \sum_{i=1}^{J} N(\alpha|\mu_i, \sigma_i^2)\end{aligned}$$

Again skill is viewed by investors as being distributed as a discrete mixture of normals and a Student-t.

I. Subpopulations of Funds

Being discrete the pairs (μ_i, σ_i^2) can be clustered together with those having the same (μ_k^*, σ_k^*), $k = 1, \ldots, K$, where $K \leq J$. These K clusters are subpopulations within the universe of mutual funds so the posterior distribution for skill is

$$\begin{aligned}[\widehat{\pi}(\alpha)|s, \mu_1^*, \sigma_1^{2*}, \ldots, \mu_K^*, \sigma_K^{2*}] &= \frac{B}{J+B} t_{\nu_0}\left(\alpha \middle| m_0, \left(\frac{\kappa_0+1}{\kappa_0}\right)\nu_0 s_0^2\right) \\ &\quad + \sum_{k=1}^{K} \frac{n_k}{J+B} N(\alpha|\mu_k^*, \sigma_k^{2*})\end{aligned}$$

where $s = (s_1, \ldots, s_J)'$ with $s_i = k$ if and only if $(\mu_i, \sigma_i) = (\mu_k^*, \sigma_k^{2*})$, and n_k is the number of funds in the kth subpopulation.

II. Subpopulations of Funds

- Since $n_k/(J+B)$ is bigger the larger n_k is, a large (small) subpopulation keeps getting larger (smaller).
- The number of subpopulations depends on the value of B. If $B = 0$ then there are no subpopulations. If $B \to \infty$ then each fund is its own subpopulation.
- Unfortunately, because of label switching the assignment vector s has no meaning. However, one can see if two funds belong to the same subpopulation; i.e., does $s_i = s_{i'}$?

I. Posterior Population Distribution of Skill

Assume the investors observe the alphas for the J mutual funds. The investors posterior population distribution for alpha is

$$
\begin{aligned}
[\widehat{\pi}(\alpha)|\alpha_1,\ldots,\alpha_J] &= \int [\widehat{\pi}(\alpha)|s,\mu_1^*,\sigma_1^{2*},\ldots,\mu_K^*,\sigma_K^{2*}] \\
&\quad \times d\pi(s,\mu_1^*,\sigma_1^{2*},\ldots,\mu_K^*,\sigma_K^{2*}|\alpha_1,\ldots,\alpha_J) \\
&= \frac{B}{B+J} t_{\nu_0}\left(\alpha \,\middle|\, m_0, \left(\frac{\kappa_0+1}{\kappa_0}\right)\nu_0 s_0^2\right) \\
&\quad + \sum_{k=1}^{K} \frac{n_k}{B+J} t_{\nu_k}\left(\alpha \,\middle|\, m_k, \left(\frac{\kappa_k+1}{\kappa_k}\right) s_k^2\right)
\end{aligned}
$$

where $\nu_k = \nu_0 + n_k$, and the kth subpopulation mean is $m_k = \frac{\kappa_0 m_0 + n_k \overline{\alpha}_k}{\nu_k}$, and variance

$$
s_k^2 = \nu_k^{-1}\left[\nu_0 s_0^2 + \sum_{i:s_i=k}(\alpha_i - \overline{\alpha}_k)^2 + \frac{\kappa_0 n_k}{\nu_k}(m_0 - \overline{\alpha}_k)^2\right].
$$

II. Posterior Populations Distribution of Skill

- The prior for G allows for subpopulations.
- Funds with similar skills have alphas that shrink towards a common subpopulation mean and variance, (μ_k^*, σ_k^{2*}) and not towards a global mean and variance.
- Because J is the number of domestic mutual funds, both existing and extinct ($J = 5,136$), and the longest mutual fund return history is small relative to J ($T_{max} = 474$), the posterior population distribution of alpha influences the posterior distributions of the α_js.
- Our investors posterior population distribution nests the JS predictive distribution of skill; i.e., $B = 0$ and $K = 1$.
- One difficulty with our investors prior DP beliefs is their initial guess for the distribution of μ_α and σ_α cannot be $G_0(\mu_\alpha, \sigma_\alpha^2) \propto 1/\sigma_\alpha^2$. G_0 has to be proper.

Sampler

Prior:

- $\pi(\beta_i, \sigma_i^2) \propto 1/\sigma_i^2$
- $G_0 \equiv$ Normal-Inverse-Gamma$(m_0, \sigma_\alpha^2/\kappa_0, \nu_0/2, \nu_0 s_0^2/2)$ where $m_0 = 0$, $\kappa_0 = 0.1$, $\nu_0 = 0.01$, and $s_0^2 = 0.01$.
- $\pi(B) =$ Gamma$(2, 30)$; i.e., $E[B] = 1/15$ and $V[B] = 1/450$.

Sampler:

1. Draw from $\beta_i, \sigma_i | r_i, \alpha_i$, for $i = 1, \dots, J$.
2. Draw from $\alpha_i | r_i, \beta_i, \sigma_i, \mu^*_{s_i}, \sigma^*_{s_i}, s_i$, for $i = 1, \dots, J$.
3. Draw (μ^*_k, σ^*_k), s and K conditional on $\alpha_1, \dots, \alpha_J$ using the DPM Polya urn sampler of West, Müller, and Escobar (1994).
4. Draw B with Escobar and West (1995).
5. 60K total draws with the first 10K discarded.

Empirical Investigation of Skill

Mutual Fund Data:

- Compounded monthly returns of 5,136 domestic equity mutual funds from January 1961 to June 2001.
- Gross monthly returns converted into annual percentage returns.
- Each fund has at least 12 months of returns.
- Funds have on average 77.3 monthly returns.

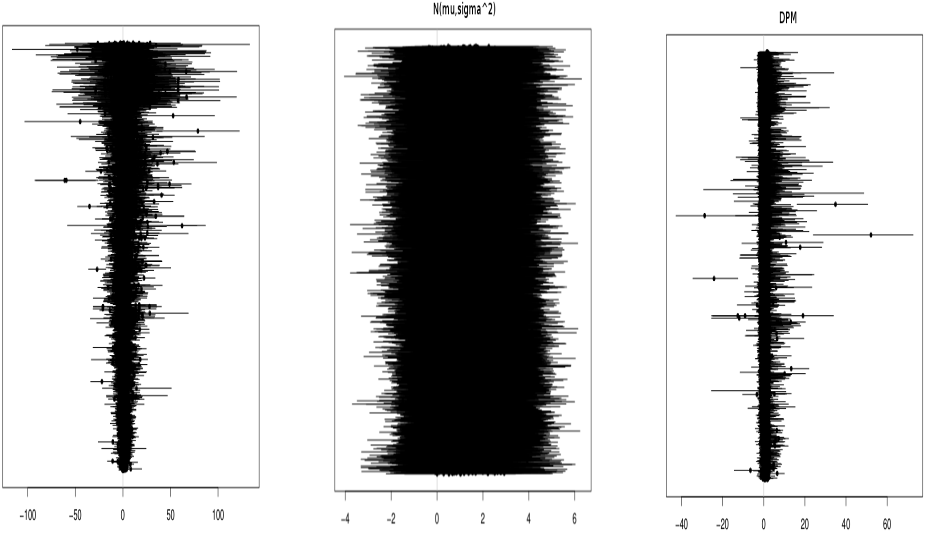
Three Investor Types:

1. Ignorant about how skill is distributed over the universe of mutual funds – DPM.
2. Assumes a normal population distribution with unknown μ_α and σ_α^2 and the prior $\pi(\mu_\alpha, \sigma_\alpha^2) \propto 1/\sigma_\alpha^2$.
3. Assumes skill is distributed over mutual funds as $\pi(\alpha) = N(0, \sigma_\alpha^2)$ where $\sigma_\alpha^2 \to \infty$; i.e., mutual funds skill are independent of one another.

Empirical Performance

- Posterior median number of subpopulations is four with a 95% HPI from three to five subpopulations.
- Posterior mean DP concentration parameter is 0.1245 with a 95% HPI of $(0.0038, 0.0261)$. So after 5136 funds there is a 0.002% chance of a new sub-population.
- 289 of the 5136 funds have negative alphas. In other words, 6% of the actively managed funds could not beat investing in a passive Fama-French, momentum portfolio.
- To add insult to injury these same 289 funds charged on average an expense fee of 1.28% a year.

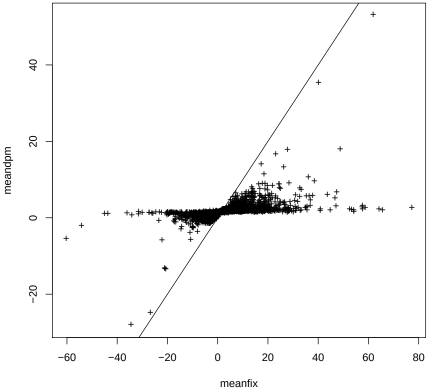
I. Alphas 95% HPI Sorted by the Fund's History Length

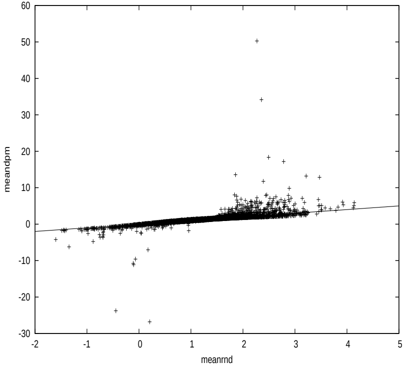


I. Alphas Sorted by the Fund's History Length

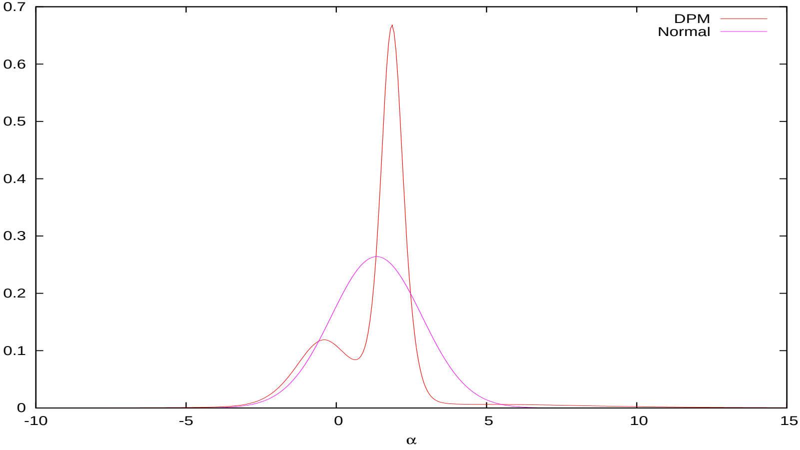
- Skill level among funds is neither idiosyncratic nor distributed according to a common population mean and variance.
- Past inference on mutual fund performance has been based on strong assumptions about the cross-sectional distribution of skill.
- One approach allows for too much variation over skill, and the other too little.
- By borrowing information from other funds belonging to the same sub-population, our performance measure provides better precision than existing approaches. This is especially true when the return history is short.
- The "are they skilled or are they just lucky?" literature of Kosowski, Timmerman Wermers and White (2006), Barras, Scaillet and Wermers (2010) and Fama and French (2010), is based on a cross-sectional distribution bootstrapped from the posterior mean of the alphas in panel (a) where a priori the population distribution does not exist.

Shrinkage


meandpm
meanfix
−60
−40
−20
0
20
40
60
80
−20
0
20
40


meandpm
meanrnd
-2
-1
0
1
2
3
4
5
-30
-20
-10
0
10
20
30
40
50
60

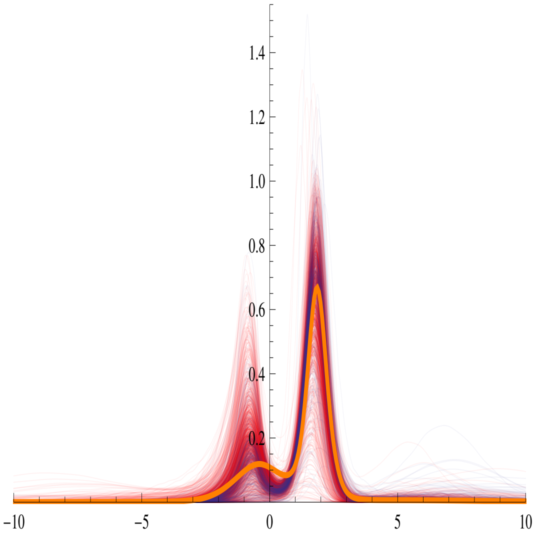
I. Population Distribution of Mutual Fund Skill



II. Population Distribution of Mutual Fund Skill

- There are three clear sub-populations in the universe of mutual fund performance.
- The cross-sectional distributions expected performance equals 1.4 percent for both the DPM and Normal prior distribution.
- Fatter tails are found in both the left, and right, hand tails of the DPM posterior predictive. Hence, performance among mutual funds can be extraordinarily good or bad.

Extraordinary Performance



Extraordinary Performance

- In general, the posterior cross-sectional distribution, with its 396,820 monthly returns from 5,136 funds, contains a wealthy amount of information about how a particular funds will perform in the future.
- For a few extraordinary funds, whose posterior distribution of alpha differs from the posterior cross-section distribution, their return history drives a sizeable wedge between their distribution and the cross-sectional distribution.
- Of the thirty-six highest skilled funds all were still in business at the end of the sample except for one (in business from 05/1988 to 01/1992).
- The oldest of these highly skilled funds opened for business in 06/1963.
- Except for one fund (opened in 1987), the start date of the other skilled funds were all after 1994.
- Potential evidence in support of Berk and Green (2004) economic equilibrium hypothesis where funds face decreasing

Conclusions

- Assumptions about how skill is distributed over the universe of mutual funds matters for infering mutual fund performance and making investment decisions.
- For example, bootstrapping the empirical distribution of skill from independent estimates of skill suggests there are more highly skilled and unskilled funds than there actually are.
- Regarding mutual fund performance, most mutual funds just cover their costs. However, there are some extraordinarly skilled and unskilled funds.
- Some truth to the economic equilibium condition of Berk and Green (2004).