How Safe are European Safe Bonds? An Analysis from the Perspective of Modern Portfolio Credit Risk Models

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Abstract

Several proposals for the reform of the euro area advocate the creation of a market in synthetic securities backed by portfolios of sovereign bonds. Most debated are the so-called European Safe Bonds or ESBies proposed by (Brunnermeier et al. 2017). Since the potential benefits of ESBies hinge on the assertion that these products can fulfill the function of a safe asset for the euro area, this paper provides a comprehensive quantitative analysis of ESBies. Our first contribution is a novel dynamic credit risk model which captures salient features of sovereign CDS spreads in the euro area. After successful calibration of our model, we study in detail the risks associated with ESBies. We discuss model-independent price bounds and the rating of ESBies, we analyse the impact of model parameters and attachment points on the size and the volatility of the credit spread of ESBies and we consider several approaches to assess the market risk of ESBies. We conclude with a brief discussion of the policy implications from our analysis.

Keywords. European Safe Bonds, European monetary union, Securitization of credit risk, Markov modulated affine models

1 Introduction

Synthetic securities backed by portfolios of sovereign bonds from the euro area have recently been proposed as a tool to improve the functioning of the European monetary union, see for instance Dombrovskis and Moscovici (2017) or Bénassy-Quéré et al. (2018). The most debated proposal is due to Brunnermeier et al. (2017). They advocate the creation of a market in so-called European Safe Bonds or ESBies. In credit risk terminology, ESBies form the senior tranche of a CDO backed by a diversified portfolio of sovereign bonds from all members of the euro area. According to Brunnermeier et al. (2017), ESBies would be standardized and issued by tightly regulated private institutions or by a public agency. The junior tranche of the underlying bond portfolio would be sold in the form of European Junior Bonds (EJBies) to investors traditionally bearing default risk, such as hedge funds or insurance companies.

Brunnermeier et al. (2017) argue that a liquid market in ESBies would enhance the stability of the euro area in a number of ways: first, it would increase the supply of safe assets in the euro area; second, it would help to break the vicious circle between bank solvency and the credit quality of sovereigns created by the fact that most euro area banks hold large amounts of risky sovereign bonds of the nation state in which they reside; third, it might reduce the distortions

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on bond markets caused by the flight-to-safety behavior of investors in crisis times. These potential benefits of ESBies hinge on the assertion that ESBies are really safe. To address this issue, Brunnermeier et al. (2017) carry out a simulation study in an one-period mixture model where defaults are independent given the aggregate state of the euro area economy. They find that, with reasonably high levels of subordination, the expected loss of ESBies is comparable to that of triple-A rated bonds. However, their model is calibrated in a fairly ad hoc manner. More importantly, Brunnermeier et al. (2017) do not study the *market risk* of ESBies (the risk of a change in the market value of these products due to changes in the credit quality of the underlying bonds or in the state of the euro area economy). Now, the bad performance of many highly rated rated senior CDO tranches during the financial crisis has shown that the market risk of such products can be substantial. Clearly, a high amount of market risk is inappropriate for a safe asset intended to serve as collateral in security market transactions, as an investment vehicle for money market funds or as a crisis-resilient store of value on the balance sheet of banks. A thorough quantitative analysis of the risks associated with ESBies is thus needed to assess if these securities can in fact perform the function of a safe asset for the euro area. This is the aim of the present paper.

We make the following contributions. We propose a novel dynamic credit risk model that captures salient features of the credit spread dynamics of euro area member states and that is at the same time fairly tractable. Such a model is a prerequisite for the analysis of the market risk associated with ESBies. In mathematical terms we consider a reduced-form model with conditionally independent default times; the hazard rate or default intensity of the different obligors is modelled by CIR-type processes whose mean-reversion level is a function of a *common* finite state Markov chain. Considering a Markov modulated mean-reversion level permits us to model different regimes, such as a crisis regime where the default intensity of all sovereigns is high and an expansionary regime where all default intensities are low. This generates default dependence in a natural way. We calibrate the model to a time series of euro area CDS spreads over the period January 2009 until September 2018, and we obtain reasonable parameter values and a good fit to the observed spreads.

Armed with these results, we turn to the risk analysis of ESBies and EJBies. We derive model-independent price bounds for these products and we shed some light on a recent proposal of S&P for the rating of ESBies (Kraemer 2017). In particular, we show that the S&P proposal is ultra-conservative in the sense that it attributes to an ESB the worst rating that is logically consistent with the ratings attributed to the euro area sovereigns. As a next step we study the robustness of the credit spread (or equivalently the risk-neutral expected loss) of ESBies and EJBies with respect to subordination levels and model parameters. In particular, we consider several parameterizations for the transition intensities of the common Markov chain, as these largely drive the default dependence in our model. It turns out that, from this perspective, ESBies are very safe already for low subordination levels (around 15%), in line with the findings of Brunnermeier et al. (2017).

We use several approaches to gauge the market risk of ESBies. First, we compute spread-trajectories for ESBies via historical simulation, using as input the calibrated trajectories of the default intensities and of the common Markov chain, and we analyse the relation between the attachment point of an ESB and the volatility of the ESB-spreads. Second, we carry out a scenario analysis and study how the risk-neutral default probability of these products is affected by changes in the underlying risk factors. To robustify our conclusions, we consider also various contagion scenarios. The results of this analysis are more nuanced. For low subordination levels and adverse scenarios (such as the case where the default of a major euro area sovereign leads to a recession in the euro area) the loss probability of ESBies can be fairly large, and spread trajectories can be quite volatile. For high subordination levels exceeding 30–35% on the other hand, ESBies remain 'safe' even in these adverse scenarios. Third, we use simulations to compute Value at Risk and Expected Shortfall for the return distribution of ESBies. For this we need to estimate the historical dynamics of the default intensities and the common Markov chain which is done via a suitable variant of the EM algorithm.

In a nutshell we conclude that, while in normal times ESBies are indeed very safe, they may become quite risky under extreme circumstances and in contagion scenarios, in particular if the attachment point is not sufficiently high. From a policy perspective, the introduction of ESBies should therefore be accompanied by appropriate policy measures to limit the economic implications of a default in the euro area, so that contagion becomes less of an issue.

Our paper is related to several strands of the literature on sovereign credit risk and CDO pricing. We begin with empirical work on the euro area CDS spreads and the sovereign debt crisis. Ang and Longstaff (2013) calibrate a multifactor affine model with systemic and country-components to euro area CDS spreads and they compare their results to US data. Ait-Sahalia, Laeven and Pelizzon (2014) fit a multivariate Hawkes model with cross and self-exciting intensities to various pairs of European sovereigns. Both papers find strong evidence for systemic sovereign default risk in the euro area, a finding which is fully in line with the economic intuition underlying the credit risk model proposed in our paper. An early contribution on CDO pricing in affine models is Duffie and Gârleanu (2001). Brigo, Pallavicini and Torresetti (2010) give an extensive discussion of CDO pricing models and their empirical properties before and during the financial crisis, see also McNeil, Frey and Embrechts (2015). Our paper is also related to work on financial innovation and markets for (quasi) safe assets such as Gennaioli, Shleifer and Vishny (2012) or Golec and Perotti (2015) and to work on euro area reform such as Bénassy-Quéré et al. (2018). Mathematical results on affine processes with Markov modulated mean reversion level can be found in Elliott and Siu (2009) and in van Beek, Mandjes, Spreij and Winands (2014)

The remainder of the paper is structured as follows. Section 2 introduces the model, in Section 3 we discuss the pricing of the relevant credit products, and Section 4 outlines how we calibrate our model to market data. Further, in Section 5 we derive model independent price bounds for ESBies and EJBies. Section 6 is devoted to a thorough analysis of the risks associated with ESBies: Section 6.1 deals with the expected loss; Sections 6.2 to 6.4 deal with the market risk of ESBies and Section 6.5 summarizes the findings from the risk analysis and discusses policy implications.

2 The Model

Throughout we consider a portfolio of J sovereigns with default times τ^j and default indicators $Y_t^j = \mathbb{1}_{\{\tau^j \leq t\}}$, $1 \leq j \leq J$, defined on a probability space $(\Omega, \mathcal{F}, \mathbb{Q})$ with filtration $\mathbb{G} = (\mathcal{G}_t)_{t\geq 0}$. $\mathbb{G}$ is the global filtration, that is all processes introduced are $\mathbb{G}$ adapted. In financial terms the σ-field $\mathcal{G}_t$ describes the information available to investors at time t. We assume that $(\Omega, \mathcal{F}, \mathbb{Q})$ supports a J-dimensional standard Brownian motion $\boldsymbol{W} = (W_t^1, \dots, W_t^J)_{t\geq 0}$ and a finite-state Markov chain X, independent of $\boldsymbol{W}$, with state space $S^X = \{1, 2, \dots, K\}$ and generator matrix $Q = (q_{kl})_{1\leq k,l\leq K}$. The chain X will be used to model transitions between K different states or regimes of the euro area economy, and for $k \neq l$, q_{kl} gives the intensity of a jump from state k

to state l. The measure $\mathbb{Q}$ is the risk-neutral measure used for the valuation of ESBies; price dynamics under the historical measure $\mathbb{P}$ are considered in Section 6.4. In the analysis of the model we also use the filtration $\mathbb{F} = (\mathcal{F}_t)_{t \geq 0}$ that is generated by the Brownian motion $\boldsymbol{W}$ and the Markov chain X.

Our default model is outlined in the following two assumptions.

A1) For all $1 \leq j \leq J$ the default time τ^j is given by $\tau^j = \inf \left\{ t \geq 0 \colon \int_0^t \gamma_s^j ds > \Theta^j \right\}$. Here $\gamma^1, \ldots, \gamma^J$ are strictly positive processes (the hazard rates of the default times), and $\Theta^1, \ldots, \Theta^J$ are independent unit-exponentially distributed random variables that are independent of the hazard rate processes $\gamma^1, \ldots, \gamma^J$. For notational convenience we introduce the vector process $\boldsymbol{\gamma} = (\gamma_t^1, \ldots, \gamma_t^J)_{t \geq 0}$.

A2) The processes $\gamma^1, \ldots, \gamma^J$ follow CIR-type dynamics with Markov modulated and time-dependent mean-reversion level, that is

$$d\gamma_t^j = \kappa^j (\mu^j(X_t) e^{\omega^j t} - \gamma_t^j) dt + \sigma^j \sqrt{\gamma_t^j} dW_t^j, \; 1 \leq j \leq J, \tag{2.1}$$

for constants $\kappa^j, \sigma^j > 0$, $\omega_j \geq 0$ and functions $\mu^j \colon S^X \to (0, \infty)$.

Assumption A1 is the standard construction of conditionally independent doubly stochastic default times, see for instance McNeil et al. (2015, Chapter 17). For small Δt the quantity $\mathbb{1}_{\{\tau^j > t\}} \gamma_t^j \Delta t$ gives the probability that firm j defaults in the period $(t, t + \Delta t]$, that is γ^j is the *default intensity* of firm j. Assumption A1 implies that $\tau^1, \ldots, \tau^J$ are independent given the realisation of the process $\boldsymbol{\gamma}$. Dependence of default events is caused by the special form of the hazard rate dynamics in A2. More precisely, the assumption that the mean-reversion levels $\mu^1, \ldots, \mu^J$ of the hazard rate processes depend on the common finite-state Markov chain X creates co-movement in the hazard rate of different sovereigns, so that, unconditionally, default times are dependent. Our setup permits also country-specific fluctuations in hazard rates; these are generated by the independent Brownian motions $W^1, \ldots, W^J$ driving the hazard rate dynamics. Time dependence in the mean reversion level is introduced via the factor $e^{\omega^j t}$, since this facilitates the calibration to single-name CDS spreads.

Following Brunnermeier et al. (2017), we mostly consider $K = 3$ states of the euro area economy. We interpret these states as expansion (state one), mild recession (state two) and strong recession (state three). This is in line with our calibration results in Sections 4. There we find that, for the vast majority of euro area members, $\mu^J(1) < \mu^j(2) < \mu^j(3)$; that is, the mean reversion level of the hazard rates of euro area members is lowest in state one and highest in a state three. A statistical analysis in Section 6.4 shows that a model of the form (2.1) with $K = 3$ states can also be used to describe the evolution of the calibrated hazard rates under the historical measure $\mathbb{P}$.

Remark 2.1. It is often claimed that the default of a euro area member might lead to *contagion*, that is, one expects the credit spreads of non-defaulted member states to jump upward in reaction to a default. This cannot be modelled in a setting with conditionally independent defaults where the hazard rate dynamics (and hence single-name credit spreads) are exogenously given and independent of the default history. Nonetheless, we prefer to work with conditional independence[3], for the following reasons. First, if appropriate measures are taken to limit the economic fallout from the default of an euro area member, conditional

[3]In Section 6.3 we relax this assumption and study the impact of contagion on the market risk for ESBies.

independence might be realistic, see Bénassy-Quéré et al. (2018) for an in-depth discussion of this point. Second, conditional independence is sufficient for a static risk analysis of ESBies. In fact, we explain in Section 5 that the whole range of arbitrage-free prices of (stylized) ESBies and EJBies consistent with observed CDS spreads can be obtained within our model if parameters are chosen appropriately. Third, without conditional independence, the price of single-name credit derivatives depends on the default state and the hazard rate of other sovereigns in the portfolio, and the calibration of the model to single-name CDS spreads is possible only for very small portfolios.

3 Credit Derivatives and ESBies

In this section we describe the payoff of credit default swaps (CDSs) and of ESBies and EJBies, and we sketch our methodology for pricing these products.

Portfolio loss. We fix a horizon $T > 0$ and a set $\mathbb{T}$ of payment dates $0 = t_0 < t_1 < \cdots < t_N = T$ which, in practical applications, usually correspond to quarterly payments. We define for $1 \leq j \leq J$ the cumulative *loss process* L^j of sovereign j by

$$L_t^j = \sum_{n=1}^{N} \mathbb{1}_{\{t_{n-1} < \tau^j \leq t_n\}} \mathbb{1}_{\{t \geq t_n\}} \delta_{t_n}^j, \quad t \in [0, T], \tag{3.1}$$

where the random variable $\delta_{t_n}^j$ gives the loss given default (LGD) of sovereign j at time t_n.[4] We assume that, given $\mathcal{F}_{t_n}$, the LGD $\delta_{t_n}^j$ is beta distributed with $E\big(\delta_{t_n}^j \mid \mathcal{F}_{t_n}\big) = \delta^j(X_{t_n})$ for a function $\delta^j : S^X \to (0, 1]$. We further assume that, given X_{t_n}, $\delta_{t_n}^j$ is independent of all other model quantities. In our parametrization the function δ^j is increasing, so that the LGD is higher in recession states. Given portfolio weights $w^j > 0$ such that $\sum_{j=1}^J w^j = 1$, we define the *portfolio loss* by

$$L_t = \sum_{j=1}^{J} w^j L_t^j, \quad t \leq T. \tag{3.2}$$

The payoff of relevant credit derivatives can be described in terms of L^j and L. In particular, we model the cash flow stream of the protection-buyer position in a CDS on sovereign j with spread x and premium payment dates $\mathbb{T}$ by

$$L_t^j - \sum_{t_n \leq t} x(t_n - t_{n-1}) \mathbb{1}_{\{\tau_j > t_n\}}, \quad 0 \leq t \leq T. \tag{3.3}$$

ESBies and EJBies. ESBies have not been issued so far, so there is no description of the payment structure of an actual product and no term sheet. Therefore, we consider stylized versions of these products. These stylized ESBies and EJBies do capture the essential features of every CDO structure, namely pooling and tranching of default risk, so they suffice to analyze the qualitative properties of ESBies. Denote by $V_T = 1 - L_T$ the normalized value of the asset pool and note that $V_T = 1$ if there are no defaults in the portfolio. The constant $\kappa \in (0, 1)$

[4] We prefer to work with (3.1) instead of the more standard definition $L_t^j = \mathbb{1}_{\{\tau_j \leq t\}} \delta^j(X_{\tau^j})$ as (3.1) is more convenient for CDS pricing. In any case, for $(t_n - t_{n-1})$ small the two definitions of L^j are close to each other.

denotes the lower (upper) attachment point of the senior (junior) tranche. Then the payoff of a stylized ESB respectively a stylized EJB at T is defined to be

$$\mathrm{ESB}_T = \min(V_T, 1-\kappa) = V_T - (V_T - (1-\kappa))^+ = (1-L_T) - (\kappa - L_T)^+\,, \tag{3.4}$$

$$\mathrm{EJB}_T = (V_T - (1-\kappa))^+ = (\kappa - L_T)^+\,. \tag{3.5}$$

In this way, the EJB bears the first 100κ percent of the loss in the portfolio, if the loss exceeds κ, the ESB is affected as well. While stylized ESBies and EJBies are path independent, in the sense that their payoff is a function of the portfolio loss at the maturity date T only, our analysis is easily extended to path dependent payoffs.

Note that, by definition, we have the following put-call-parity-type relation for the payoff of a stylized ESB and a stylized EJB with identical attachment point κ

$$\mathrm{ESB}_T + \mathrm{EJB}_T = V_T \text{ and hence } \mathrm{ESB}_T = (1-L_T) - \mathrm{EJB}_T\,. \tag{3.6}$$

Pricing Methodology. For simplicity, we assume that the risk-free short rate is constant and equal to $r \geq 0$. We introduce the money market account $B_{t,s} = \exp(r(s-t))$, $s > t$, so that $B_{t,s}^{-1}$ is the discount factor at time t for a payoff due at time s. We use standard risk-neutral valuation for the pricing of credit derivatives. Hence the price at t of any integrable $\mathcal{G}_s$ measurable contingent claim H is equal to $H_t = E\left(B(t,s)^{-1}H \mid \mathcal{G}_t\right)$, where the expectation is taken under the risk-neutral measure $\mathbb{Q}$.

Our main tool for computing prices of credit derivatives is the following extension of the extended Laplace transform for CIR processes, see Duffie, Pan and Singleton (2000), to the case of a Markov modulated mean-reversion level. A related result was derived in Elliott and Siu (2009) for the case a single CIR-type process, see also van Beek et al. (2014)

Proposition 3.1. *Denote by $\mathbb{F} = (\mathcal{F}_t)_{t\geq 0}$ the filtration generated by the Brownian motion $\boldsymbol{W}$ and the Markov chain X. Consider vectors $\boldsymbol{a}, \boldsymbol{u} \in \mathbb{R}^J_+$ and a function $\xi : S^X \to \mathbb{R}$. Fix some horizon date $s \leq T$. Then it holds that for $0 \leq t < s$*

$$E\left(\xi(X_s)\exp\left(-\int_t^s \boldsymbol{a}'\boldsymbol{\gamma}_\theta d\theta\right) e^{-\boldsymbol{u}'\boldsymbol{\gamma}_s} \mid \mathcal{F}_t\right) = v(t, X_t)\exp\left(\boldsymbol{\beta}(s-t,\boldsymbol{u})'\boldsymbol{\gamma}_t\right). \tag{3.7}$$

Here $\boldsymbol{\beta}(\cdot,\boldsymbol{u}) = (\beta_1(\cdot,\boldsymbol{u}),\ldots,\beta_J(\cdot,\boldsymbol{u}))'$ and the functions $\beta_j(\cdot,\boldsymbol{u})$, $1 \leq j \leq J$, solve the Riccati equation

$$\partial_t\beta_j(t,\boldsymbol{u}) = -\kappa^j\beta_j(t,\boldsymbol{u}) + \frac{1}{2}(\sigma^j)^2\beta_j^2(t,\boldsymbol{u}) - a_j, 0 < t \leq s\,, \tag{3.8}$$

with initial condition $\boldsymbol{\beta}(0,\boldsymbol{u}) = -\boldsymbol{u}$. Moreover, with $\boldsymbol{v}(t) = \big(v(t,1),\ldots,v(t,K)\big)'$, the function $v\colon [0,s]\times S^X \to \mathbb{R}$ satisfies the linear ODE system

$$-\frac{d}{dt}\boldsymbol{v}(t) - \operatorname{diag}\left(\bar{\mu}_1(t),\ldots,\bar{\mu}_K(t)\right)\boldsymbol{v}(t) = Q\boldsymbol{v}(t), \quad \text{on } [0,s], \tag{3.9}$$

with terminal condition $\boldsymbol{v}(s) = \boldsymbol{\xi}$ and with $\bar{\mu}_k(t) = \sum_{j=1}^J e^{\omega^j t}\kappa^j\mu^j(k)\beta_j(s-t,\boldsymbol{u})$.

The proof is given in Appendix A. The functions $\beta_j(t,\boldsymbol{u})$ are known explicitly, see for instance Filipovic (2009) for details. Essentially, Proposition 3.1 shows that computing the extended Laplace transform of $\boldsymbol{\gamma}$ is not much more complicated than in the classical case of independent CIR processes; the only additional step is to solve the K-dimensional linear ODE system (3.9) for the function $\boldsymbol{v}(t)$, which is straightforward to do numerically.

Next, to illustrate the use of Proposition 3.1, we consider the pricing of a survival claim and of a CDS on sovereign j.

Survival claim. The payoff of a survival claim on sovereign j with maturity date s and payoff function $f\colon S^X \to \mathbb{R}$ is of the form $\mathbb{1}_{\{\tau^j > s\}} f(X_s)$. Using standard results on doubly stochastic default times, the price of this claim at time $t \leq s$ is

$$E\left(B_{t,s}^{-1} \mathbb{1}_{\{\tau^j > s\}} f(X_s) \mid \mathcal{G}_t\right) = \mathbb{1}_{\{\tau^j > t\}} B_{t,s}^{-1} E\left(e^{-\int_t^s \gamma_s^j ds} f(X_s) \mid \mathcal{F}_t\right),$$

and the expectation on the right can be computed from Proposition 3.1 with $\boldsymbol{a} = \boldsymbol{e}^j$, $\boldsymbol{u} = \boldsymbol{0}$ and $\xi = f$.

Credit default swap. We briefly discuss CDS pricing in our setup, since this is crucial for model calibration. From the payoff description (3.3), pricing a CDS contract amounts to computing the conditional expectation

$$E\left(\sum_{n=1}^{N} B_{t,t_n}^{-1} \mathbb{1}_{\{\tau_j \in (t_{n-1}, t_n]\}} \delta_{t_n}^j - \sum_{n=1}^{N} x(t_n - t_{n-1}) B_{t,t_n}^{-1} \mathbb{1}_{\{\tau_j > t_n\}} \mid \mathcal{G}_t\right). \tag{3.10}$$

Denote by V_t^{prem} and V_t^{def} the present value of the premium and the default leg, that is

$$\begin{aligned} V_t^{\text{prem}}(x) &= \sum_{n=1}^{N} B_{t,t_n}^{-1} x(t_n - t_{n-1}) E\left(\mathbb{1}_{\{\tau_j > t_n\}} \mid \mathcal{G}_t\right), \\ V_t^{\text{def}} &= \sum_{n=1}^{N} B_{t,t_n}^{-1} E\left(\mathbb{1}_{\{\tau_j \in (t_{n-1}, t_n]\}} \delta_{t_n}^j) \mid \mathcal{G}_t\right) \\ &= \sum_{n=1}^{N} B_{t,t_n}^{-1} E\left(\mathbb{1}_{\{\tau_j \in (t_{n-1}, t_n]\}} \delta^j(X_{t_n}) \mid \mathcal{G}_t\right). \end{aligned} \tag{3.11}$$

To obtain (3.11), we have used the fact that the default leg of the CDS is linear in the loss given default, so that we can replace $\delta_{t_n}^j$ with its conditional expectation. The premium leg is simply the sum of survival claims. The evaluation of (3.11) is more involved, and we now show how this can be achieved via Proposition 3.1. Fix any two consecutive payment dates t_{n-1}, t_n of $\mathbb{T}$ and assume w.l.o.g. that $t \leq t_{n-1}$. Since $\mathbb{1}_{\{t_{n-1} < \tau^j \leq t_n\}} = \mathbb{1}_{\{\tau^j > t_{n-1}\}} - \mathbb{1}_{\{\tau^j > t_n\}}$, we can write the term $E\left(\mathbb{1}_{\{\tau_j \in (t_{n-1}, t_n]\}} \delta_j(X_{t_n}) \mid \mathcal{G}_t\right)$ in the form

$$E\left(\mathbb{1}_{\{\tau^j > t_{n-1}\}} \delta^j(X_{t_n}) \mid \mathcal{G}_t\right) - E\left(\mathbb{1}_{\{\tau^j > t_n\}} \delta^j(X_{t_n}) \mid \mathcal{G}_t\right). \tag{3.12}$$

The second term in (3.12) is a survival claim. By iterated conditional expectations, we get that the first term is equal to

$$E\left(\mathbb{1}_{\{\tau^j > t_{n-1}\}} E\left(\delta^j(X_{t_n}) \mid \mathcal{G}_{t_{n-1}}\right) \mid \mathcal{G}_t\right). \tag{3.13}$$

Since X is Markov, it holds that $E\left(\delta^j(X_{t_n}) \mid \mathcal{G}_{t_{n-1}}\right) = v^\delta(t_{n-1}, X_{t_{n-1}})$ for a suitable function $v^\delta\colon [0, t_n] \times S^X \to \mathbb{R}$ (given by the solution of an ODE system), so (3.13) reduces to computing $E\left(\mathbb{1}_{\{\tau^j > t_{n-1}\}} v^\delta(t_{n-1}, X_{t_{n-1}}) \mid \mathcal{G}_t\right)$, which is a standard pricing problem for a survival claim.

ESBies. Let $\boldsymbol{L}_t = (L_t^1, \ldots, L_t^J)$. The price of an ESB at time $t \in \{t_0, t_1, \ldots, t_N\}$ is given by

$$E\left(B_{t,T}^{-1}((1 - L_T) - (\kappa - L_T)^+) \mid \mathcal{G}_t\right) =: h^{\mathrm{ESB},\kappa}(t, X_t, \boldsymbol{\gamma}_t, \boldsymbol{L}_t) \tag{3.14}$$

for a suitable function $h^{\mathrm{ESB},\kappa}$. This follows from the fact that the processes $\left(X_{t_n}, \boldsymbol{\gamma}_{t_n}, \boldsymbol{L}_{t_n}\right)_{n=0}^N$ are jointly Markov; we omit the details. Similarly, the price of an EJB is given by

$$h^{\mathrm{EJB},\kappa}(t, X_t, \gamma_t, \boldsymbol{L}_t) := E\left(B_{t,T}^{-1}(\kappa - L_T)^+) \mid \mathcal{G}_t\right). \tag{3.15}$$

In order to evaluate the function $h^{\mathrm{EJB},\kappa}$ we use Monte Carlo simulation. For the computation of the function $h^{\mathrm{ESB},\kappa}$ we use that $h^{\mathrm{ESB},\kappa} = E\left(B_{t,T}^{-1}(1 - L_T) \mid \mathcal{G}_t\right) - h^{\mathrm{EJB},\kappa}$ and we compute the expected discounted portfolio loss analytically using Proposition 3.1.

4 Calibration

4.1 Data and calibration design

The available data consist of weekly CDS spread quotes from ICE data services for ten euro area sovereigns and times-to-maturity equal to 1, 2, 3, 4 and 5 years over the period January 7, 2009 until September 3, 2018, giving rise to 510 observation dates. The sovereigns used in our analysis are Austria (AUT), Belgium (BEL), Germany (DEU), Spain (ESP), Finland (FIN), France (FRA), the Republic of Ireland (IRL), Italy (ITA), the Netherlands (NLD) and Portugal (PRT), making up more than 90% of the eurozone GDP in 2018. Table 4 reports summary statistics (sample mean, sample standard deviation, minimum and maximum) of the CDS spreads, and their most recent Standard & Poor's credit-rating. Average spreads vary considerably across countries and, with the exception of Ireland, the term structures of the average spreads is upward sloping.

We calibrate the model via a loss function approach, using a set of modern optimization algorithms. For a given sovereign j, we denote by $\mathrm{cds}_{t,u}^j$ the market CDS spread at time t with time to maturity u and by $\widehat{\mathrm{cds}}_{t,u}^j$ the corresponding model spread. We work with three states of X and, in order to reduce the dimension of the parameter space, we fix the mean function of the state-dependent LGD δ^j at the outset. The distinct values for δ^j can be found in Table 5 in the appendix. Moreover, we use the EONIA at date t as a proxy for r_t. Hence the model spread at time t is a function of γ_t^j and X_t and of the parameters (Θ^j, σ^j, Q) with $\Theta^j = (\mu^j(1), \mu^j(2), \mu^j(3), \kappa^j, \omega^j)$, that is

$$\widehat{\mathrm{cds}}_{t,u}^j = \widehat{\mathrm{cds}}(u, \gamma_t^j, \Theta^j, \sigma^j, Q, X_t).$$

We use $s_0 < s_1 < \cdots < s_M$ to denote the observation dates and we write $\{\boldsymbol{\gamma}_{s_m}\} = \{\boldsymbol{\gamma}_{s_0}, \ldots, \boldsymbol{\gamma}_{s_M}\}$ and $\{X_{s_m}\} = \{X_{s_0}, \ldots, X_{s_M}\}$. We define for sovereign j and observation date s_m the quadratic loss function

$$l_{s_m}^j(\gamma_{s_m}^j, \Theta^j, \sigma^j, Q, X_{t_m}) = \sum_{u \in \mathcal{T}} \left(\mathrm{cds}_{s_m,u}^j - \widehat{\mathrm{cds}}(u, \gamma_{s_m}^j, \Theta^j, \sigma^j, Q, X_{s_m})\right)^2,$$

where $\mathcal{T}$ denotes a set of different times to maturity. We determine the model parameters and the realized trajectories $\{\boldsymbol{\gamma}_{s_m}\}$ and $\{X_{s_m}\}$ by minimizing the global calibration error $\sum_{m=1}^M \sum_{j=1}^J l_{s_m}^j(\gamma_{t_m}^j, \Theta^j, \sigma^j, Q, X_{t_m})$. For this we use an iterative approach, which is described in detail in Appendix B.

4.2 Results

We implement this calibration methodology on the full time series of available CDS data, using maturities of one and five years ($\mathcal{T} = \{1, 5\}$). The reason for this choice is that one-year CDS spreads are particularly informative regarding the current value of the hazard rates and that five-year CDS are the most liquid. To assess the quality of the calibration, we report in Table 4.2 the *root mean squared errors* (RMSE). With the exception of Spain, pricing errors for the one-year spread are extremely small for all countries. For a maturity of five years, pricing errors are somewhat larger but still fairly small compared to the range of the corresponding data (see Table 4). The highest pricing errors arise for the countries with high average spreads. The quality of the calibration is illustrated further in Figure 7 in Appendix B, where we plot the time series of CDS spreads together with the model prices and the absolute pricing errors for the Germany and Italy. Summarizing, we conclude that the calibration provides a very good fit.

Mat.	AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
					Pricing Errors (RMSE)					
1	6.36	8.70	4.73	39.72	3.58	6.75	4.58	0.45	2.90	1.49
5	15.58	15.30	9.26	45.41	6.73	13.07	40.10	34.76	10.60	66.56

Table 1: Calibration error in basis points for maturities ofone and five years.

Tables 2 and 3 report the parameter values resulting from the calibration. First, note that the ordering of the mean-reversion levels is in line with the interpretation of the states of X as expansion, mild and strong recession: it holds that $\mu^j(1) < \mu^j(2) < \mu^j(3)$.[5] This ordering of the long-term means is clear evidence that there is strong comovement in the market's perception of the credit quality of euro area members.

The mean reversion speed κ^j is quite low for all countries, and for four of them (Austria, Belgium, Finland and France) it is equal to the exogenously imposed lower bound of 0.1. Consequently, market participants expect idiosyncratic credit shocks to have a long-lasting effect across the term structure of CDS spreads. The motivation for including the parameter ω^j is to better capture the upward sloping term structure of most of the CDS series. In fact, for $\omega^j = 0$, one would obtain negative values for κ^j — a common phenomenon also reported e.g. in Ang and Longstaff (2013). Table 3 reports the estimate of the generator matrix Q. Note that, for the estimated Q, transitions to non-neighbouring states have zero probability.

Figure 1 plots the square root of the calibrated hazard rates[6] together with the calibrated trajectory of the Markov chain X. The process X remains in state one for most of the sample period, the only exceptions occur at the height of the European sovereign debt crisis from mid-2010 until late 2013, when the chain visits states two and three before settling in state one again. In general, the paths of the hazard rates are in line with the movement of the chain. Exceptional individual events such as the rise of the Portuguese hazard rates at the beginning of 2016 or the sudden upward movement of Italian rates during mid-2018 are of idiosyncratic nature.

[5]The only exception is Germany with $\mu(1) \geq \mu(2)$. This reverse ordering is easily explained by Germany's prominent role as the eurozone's safe haven in times of financial distress.

[6]We plot $\sqrt{\gamma^j}$ as this is the natural scale for a CIR process.

Param.	AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
$\mu(1)$	0.0049	0.0044	0.0027	0.0053	0.0047	0.0051	0.0177	0.0710	0.0045	0.0656
$\mu(2)$	0.0049	0.0128	0.0001	0.0189	0.0048	0.0085	0.0746	0.0727	0.0049	0.2030
$\mu(3)$	0.0424	0.0486	0.0338	0.0558	0.0209	0.0502	0.2498	0.4099	0.0458	0.2115
κ	0.1000	0.1000	0.1076	2.6879	0.1000	0.1000	0.1920	0.1215	0.1197	0.1181
ω	0.1730	0.1924	0.1534	0.0826	0.1592	0.1916	0.0004	0.0011	0.0994	0.0219
σ	0.1447	0.1152	0.0872	0.2472	0.0639	0.1075	0.1994	0.2113	0.0925	0.3162

Table 2: Calibration results: parameters of hazard rate dynamics .

	State 1	State 2	State 3
State 1 (expansion)	−0.1421	0.1421	0.0000
State 2 (mild recession)	0.5843	−1.1685	0.5843
State 3 (strong recession)	0.0000	0.9630	−0.9630

Table 3: Calibration results: generator matrix Q of X.

5 Model-independent price bounds for ESBies and EJBies

In this section, we determine model-independent price bounds for stylized ESBies and EJBies under the assumption that the expected loss of each sovereign in the portfolio is fixed. Moreover, we determine the worst-case default scenario associated with the price bounds and we relate this to a recent proposal for the rating of ESBies by the rating agency S&P, see Kraemer (2017). The expected-loss constraints can be viewed as a stylized way of calibrating the model to given CDS- or bond spreads.

We drop the time index and consider a portfolio with loss $L = \sum_{j=1}^{J} w^j \delta^j Y^j$ with deterministic exposures $w^j > 0$ summing to one and with LGD $\delta^j \in [0,1]$, $1 \le j \le J$. We do not specify the distribution of δ^j at this point. Let $L^j = \delta^j Y^j$. We assume that the expected loss of each sovereign is fixed, that is $E\left(L^j\right) = \bar{\ell}^j$ for some $\bar{\ell}^j \in (0,1)$. The payoff of stylized ESBies and and EJBies is a function of the random vector $\boldsymbol{L} = (L^1, \dots, L^J)'$, with values in $[0,1]^J$. More precisely, with $L = \sum_{j=1}^{J} w^j L^j$, it holds that $\text{EJB} = \big(\kappa - L\big)^+$ and $\text{ESB} = 1 - L - \text{EJB}$. Hence, the value of these contracts depends only on the distribution of $\boldsymbol{L}$ on $[0,1]^J$. Denote by $\mathcal{Q}$ the set of all probability measures π on $[0,1]^J$ such that $E^\pi(L^j) = \bar{\ell}^j$ for all $1 \le j \le J$; that is, $\mathcal{Q}$ is the set of all distributions of $\boldsymbol{L}$ that are consistent with the expected-loss constraints. Finding a lower bound on the value of ESBies corresponds to finding a probability measure $\pi^* \in \mathcal{Q}$ such that for all $\kappa \in [0,1]$ and all $\pi \in \mathcal{Q}$

$$E^{\pi^*}\Big(1 - L - \big(\kappa - L\big)^+\Big) \le E^{\pi}\Big(1 - L - \big(\kappa - L\big)^+\Big); \tag{5.1}$$

π^* will be called the *worst case distribution.* Since for $\pi \in \mathcal{Q}$ it holds that $E^\pi(L) = \sum_{j=1}^{m} w^j \bar{\ell}^j$, π^* equivalently solves the problem of maximizing the EJB-value $E^\pi\big((\kappa - \sum_{j=1}^{m} w^j L^j)^+\big)$ over all $\pi \in \mathcal{Q}$.

Proposition 5.1. *Assume that the sovereigns are ordered in such a way that* $\bar{\ell}^1 \le \bar{\ell}^2 \le \dots \bar{\ell}^J \le 1$*, that is index 1 corresponds to the 'strongest' sovereign, index 2 to the 'second-strongest' and so on. Then*

(i) The worst-case distribution $\pi^* \in \mathcal{Q}$ *is discrete and of the form*

$$\pi^*\big((1,\dots,1)\big) = \bar{\ell}^1,\ \pi^*\big((0,1,\dots,1)\big) = \bar{\ell}^2 - \bar{\ell}^1, \cdots,\ \pi^*\big((0,\dots,0,1)\big) = \bar{\ell}^J - \bar{\ell}^{J-1},$$
$$\pi^*\big((0,\dots,0)\big) = 1 - \bar{\ell}^J\,.$$

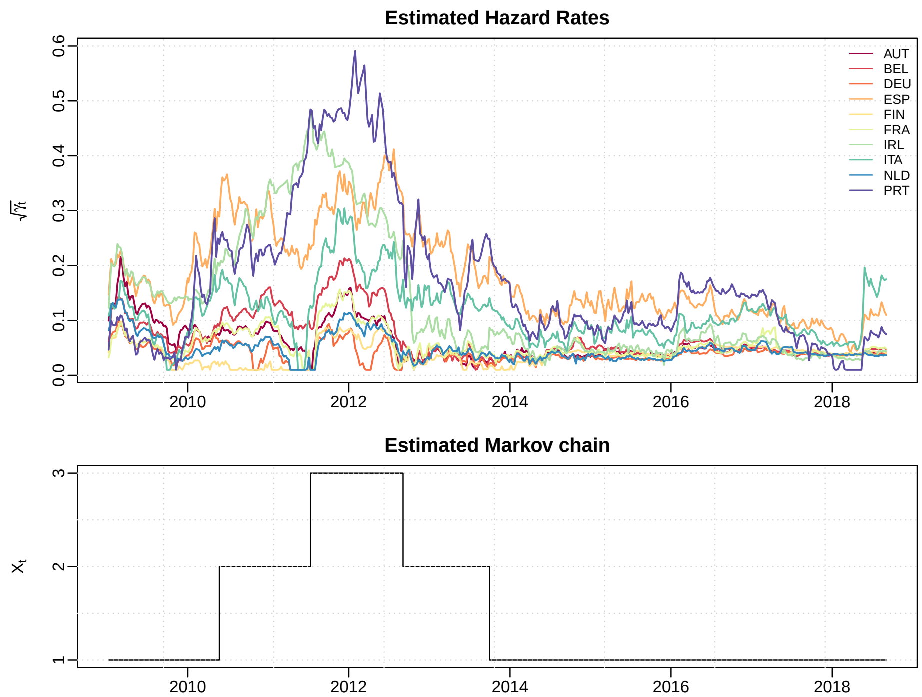

Figure 1: Time series plots of the estimated hazard rates and the calibrated Markov chain. Note that we graph $\sqrt{\gamma_t}$.

The minimal ESB price respectively the maximal EJB price consistent with the expected-loss constraints is given by $E^{\pi^*}\big(1 - L - (\kappa - L)^+\big)$ *respectively by* $E^{\pi^*}\big((L-\kappa)^+\big)$.

(ii) Given a random variable $U \sim \mathcal{U}([0,1])$*, let* $L_j^* = 1_{\{U > 1 - \bar{\ell}^j\}}$, $1 \le j \le J$*. Then the random vector* $(L_1^*, \dots, L_J^*)'$ *has distribution* π^**, that is under the worst-case distribution the random variables* $L^1, \dots, L^j$ *are* comonotonic.

The crucial point of the proposition is the fact that the dependence structure of $(L^1, \dots, L^J)'$ that minimizes the value of ESBies (or equivalently that maximizes the value of EJBies) is comonotonicity, see McNeil et al. (2015, Chapter 7) for details. A fully worked-out proof can be found in Appendix A.

We now discuss several economic implications of the proposition. First note that, under π^*, the LGD of all sovereigns is almost surely equal to one and $\bar{\ell}^j = \pi^*(Y^j = 1)$. Moreover, π^* maximizes the probability of large default "clusters": first, under π^* the event where all sovereigns default has probability $\bar{\ell}^1$. Since for $\pi \in \mathcal{Q}$,

$$\pi(L^1 = 1, \dots, L^J = 1) \le \pi(L^1 = 1) \le E^{\pi}(L^1) = \bar{\ell}^1 \,,$$

this is the maximum value possible. Next, under π^* the event where all sovereigns except the strongest default has probability $\bar{\ell}^2 - \bar{\ell}^1$. It is easily seen that this is the maximum possible value given the expected-loss constraints and the probability attributed to the first cluster (the cluster where all sovereigns default). Similarly, the probability of the $(n+1)$-th cluster, where all but the strongest n sovereigns default, is maximal given the probability attributed to the first n clusters.

In the context of our default model, the form of π^* suggests that, for a given expected-loss level of the sovereigns, an ESB is more risky if one chooses high values for $\mu^1(K), \dots \mu^J(K)$ (the mean reversion levels of the default intensity in the recession state) and if the LGD is high in the recession state K. At the same time, the generator matrix has to be parameterized in such a way that the recession state is visited relatively infrequently in order to satisfy the expected-loss constraints. This intuition underlies the construction of the *crisis scenarios* in the numerical experiments reported in the next section. Note that it is possible to approximate the price bounds for ESBies and EJBies within the class of models introduced in Section 2; a precise construction is given in Section A.3 in Appendix A.

The default mechanism in (ii) has the following interpretation. With $V = 1 - U$ it holds that $L^j = 1_{\{V < \bar{\ell}^j\}}$. The random variable V can be viewed as a measure for the strength of the euro area economy; sovereign j defaults if V is below its default threshold $\bar{\ell}^j$. Note that this construction implies that sovereigns default exactly in the order of their credit quality; in particular, it is not possible that sovereign j defaults and the "weaker" sovereign $i > j$ survives. The fact that, under π^*, the L^j are comonotonic (perfectly dependent) random variables implies that in the worst-case scenario there is no diversification between the sovereigns in the portfolio.

On the"weak-link approach" of S&P. In a recent technical document, Kraemer (2017) discusses how the rating agency S&P would determine a rating for ESBies and EJBies. The proposed methodology is termed *weak-link approach* and it works as follows: assume that the sovereigns are ordered according to their rating, so that sovereign one has the best rating and sovereign J has the worst rating and consider an ESB with attachment point κ. Define

$$j^* = \min\{1 \le j \le J \colon \sum_{i=j}^{J} w^i \ge \kappa\}.$$

Then under the weak-link approach the ESB is assigned the rating of sovereign j^*.

The assumption underlying this approach is that "sovereigns will default in the order of their ratings, with lowest rated sovereigns defaulting first" (Kraemer 2017, Page 4), so that the ESB incurs a loss as soon as the sovereign j^* defaults (in their analysis, S&P assumes that the LGD of all sovereigns is equal to one). The default mechanism underlying the weak-link approach of S&P is obviously identical to the construction of the loss variables L_j^* in Proposition 5.1(ii). We conclude that the weak-link methodology proposed by Kraemer (2017) attributes to an ESB the worst rating that is logically consistent with the ratings of the individual euro area sovereigns. In particular, diversification effects between euro area members are ignored completely.

6 Risk analysis

After the successful calibration of our model, we are now in a position to analyze the risks associated with ESBies. We begin with a short overview. In Section 6.1 we compute the risk-neutral expected loss (or equivalently the credit spread) of ESBies as a function of the attachment point κ for different parameter sets. We consider a base parameter set corresponding largely to the parameters obtained in the model calibration of Section 4, two crisis sets with higher default correlation, and the worst case loss distribution from Proposition 5.1.

The subprime credit crisis has shown that the expected loss at maturity gives only limited information regarding the riskiness of tranched credit products such as ESBies. In fact, the market value of AAA-rate senior tranches of mortgage backed securities (MBS) fell sharply during the crisis (some were even downgraded), creating huge losses for many MBS investors. To analyze if ESBies can perform all functions of a safe asset, we thus need to take a closer look at the associated market risk. We do this in several ways. First, we use a historical simulation approach and compute credit spread trajectories of ESBies for different attachment points, using as input the calibrated trajectories $\{X_{s_m}\}$ and $\{\boldsymbol{\gamma}_{s_m}\}$ from Section 4. This analysis gives useful information on the relation between κ and the volatility of ESB credit spreads. Second, many potential ESB investors, such as managers of money market funds, are extremely risk averse so that "behavior in (quasi) safe asset markets may be subject to sudden runs when new information suggests even a minimal chance of a loss" (Golec and Perotti 2015). In Section 6.3 we therefore study how the risk-neutral *loss probability* $\mathbb{Q}(L_T > \kappa)$ of ESBies is affected by changes in the underlying risk factors. To guard against model misspecification and to incorporate stylized facts regarding investor behavior on markets for safe assets, we include various contagion scenarios into this analysis. Third, in Section 6.4 we use simulations to study measures of market risk based on distribution of the negative returns of ESBies. In fact, most standard risk measures, such as Value at Risk or Expected Shortfall, are statistics of that distribution, see for instance McNeil et al. (2015).

Throughout this section we consider three possible states of the Markov chain X (i.e. $K = 3$), ESBies with a time to maturity of five years and, for simplicity, a risk free interest rate $r = 0$. Following Brunnermeier et al. (2017), we choose the portfolio weights w^j according to the GDP proportions within the euro area; numerical values are given in Table 6 in Appendix B.

6.1 Expected Loss of ESBies

In order to make the prices of ESBies with different attachment points κ comparable, we consider normalized ESBies with payoff $\frac{1}{1-\kappa}\min(V_T, 1-\kappa)$, so that the payoff of a normalized ESB is equal to one if there is no default, i.e. for $L_T \leq \kappa$. Moreover, we introduce the risk-neutral *expected tranche loss*

$$\ell^{\mathrm{ESB},\kappa}(0, X_0, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,;\, Q, \boldsymbol{\mu}) = 1 - \frac{1}{1-\kappa} h^{\mathrm{ESB},\kappa}(0, X_0, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,;\, Q, \boldsymbol{\mu})\,. \tag{6.1}$$

Here $\boldsymbol{\mu} = \{\mu^j(k)),\ 1 \leq k \leq 3,\ 1 \leq j \leq J\}$, Q is the generator matrix of X and the function $h^{\mathrm{ESB},\kappa}(t, X_t, \boldsymbol{\gamma}_t, \boldsymbol{L}_t\,;\, Q, \boldsymbol{\mu})$ gives the price of an ESB with attachment point κ at time t, see equation (3.14). We have made the parameters Q and $\boldsymbol{\mu}$ explicit in (6.1) since we want to study how variations in their values affect the expected loss of ESBies. Note that we may interpret the annualized expected loss $\frac{1}{T}\ell^{\mathrm{ESB},\kappa}$ as *credit spread* $c^{\mathrm{ESB},\kappa}(0,T)$ of a normalized ESB with attachment point κ. In fact, since $r = 0$ and since for x close to one $\ln x \approx x - 1$, it holds that

$$c^{\mathrm{ESB},\kappa}(0,T) = \frac{-1}{T}\ln\left(\frac{1}{1-\kappa}h^{\mathrm{ESB},\kappa}\right) \approx \frac{1}{T}\ell^{\mathrm{ESB},\kappa}\,.$$

Parameters. As before, we work with $K = 3$ states of X. We use the volatility parameters σ^j and the calibrated trajectories $\{\boldsymbol{\gamma}_{s_m}\}$ and $\{X_{s_m}\}$ obtained in Section 4. In our numerical experiments, we consider three different parameter sets and the worst case distribution from Proposition 5.1. In the *base parameter set* we use the generator matrix from Section 4. We take $\omega^j = 0$ and calibrate $\boldsymbol{\mu}$ and κ^j to the full CDS term structure at the valuation date, so that the parameterized model accurately reflects the market's expectation at that date.[7]

The generator matrix Q is hard to calibrate from historical data, essentially since products depending on the default correlation of euro area countries are not traded. To deal with the ensuing model risk, we introduce two *crisis parameter sets*. In these parameterizations the recession state (state three) occurs less frequently than under the base parametrization, but if it occurs default intensities are (on average) substantially larger than for the base parameter set. To achieve this, we consider two generator matrices $\widetilde{Q}_1$ and $\widetilde{Q}_2$ chosen such that, on average, X spends less time in state three than under the base parametrization. The corresponding mean reversion levels $\widetilde{\boldsymbol{\mu}}_1$ and $\widetilde{\boldsymbol{\mu}}_2$ and are determined from the constraint that the expected loss $E(L_T^j)$ is identical for all parameter sets; this typically leads to $\mu^j(3) < \widetilde{\mu}_1^j(3) < \widetilde{\mu}_2^j(3)$. The entries of $\widetilde{Q}^1$, $\widetilde{Q}^2$ are provided in Table B.4.

Results. In the top panel of Figure 2 we graph the average expected loss[8] of ESBies over the period from 2014 to September 2018 as a function of the threshold κ. We do this for the base parametrization, the two crisis parameterizations and the worst case distribution from Proposition 5.1. The scale for the y-axis is logarithmic and values are given in percentage points. In addition, we consider AAA- and A- rated sovereigns (DEU, NLD and IRL, ESP, respectively) and compute the 1%- and 99%-quantile of the risk-neutral expected loss over the period from 2014 to September 2018. Those quantiles form the boundaries of the colored areas in Figure 2; they are supposed to give an indication of the credit quality for the ESBies on a rating scale.[9]

From Figure 2 we draw the following conclusions. First, the average risk-neutral expected loss of ESBies is indeed small. For example, the average expected loss corresponding to the proposed attachment point of 0.3 is below 0.1%. Most strikingly, except for the worst case distribution, the average expected loss of ESBies with thresholds of 0.15 or higher is well below the lower bound of the AAA-region. Second, the expected loss is lowest for the base parameters, followed by crisis parameterizations 1 and 2; this is fully in line with the economic intuition underlying the construction of these parameter sets. Third, the expected loss for the worst case distribution (which is highest by construction) is substantially higher than the expected loss in the crisis parameterizations, underlining the fact that the worst case distribution, and the associated weak-link approach of Kraemer (2017), is extremely conservative. Nonetheless, for $\kappa > 0.25$ the average expected loss for the worst case distribution is still comparable in size to that of AAA-rated sovereigns. Fourth, the expected loss of an ESB is decreasing approximately at an exponential rate in κ in all four parameter sets (recall that we use a logarithmic scale for the y-axis). Summarizing, these findings show that an investor willing to hold ESBies with an attachment point of 0.15 or higher until maturity faces little risk of default-induced losses, which is in agreement with the analysis of Brunnermeier et al. (2017).

The bottom panel of Figure 2 shows the average expected loss of EJBies for varying at-

[7]The calibration in Section 4, on the other hand, yields a fixed set of parameters giving a reasonable fit throughout the entire observation period. This provides evidence for the good performance of our model in explaining market data, but is of course subject to small pricing errors at any given date.

[8]Here the term "average" refers to the average over observation dates, but with a fixed time to maturity of five years, that is we plot the function $\kappa \mapsto \frac{1}{M}\sum_{m=1}^{M} \ell^{\mathrm{ESB},\kappa}(0, X_{s_m}, \boldsymbol{\gamma}_{s_m}, \mathbf{0}\,;\, Q, \boldsymbol{\mu})$.

[9]We stress that these indicative ratings should not be taken as actual ratings of ESBies, since they are computed from risk-neutral expected losses and not from historical ones, and since a rating is more than a mechanical mapping of expected loss to some rating scale.

tachment points. With five-year expected loss levels ranging from 6% to around 15% (and hence annualized credit spreads between 1.2% and 3%) the risk of EJBies is comparable to that of lower-quality eurozone sovereigns. Comparing expected loss of ESBies and EJBies, we see that, in line with the proposal of Brunnermeier et al. (2017), EJBies bear the bulk of the credit risk associated to the eurozone sovereigns. Note that the reverse ordering of the lines in the two panels of Figure 2 is an immediate consequence of the put-call parity relation (3.6).

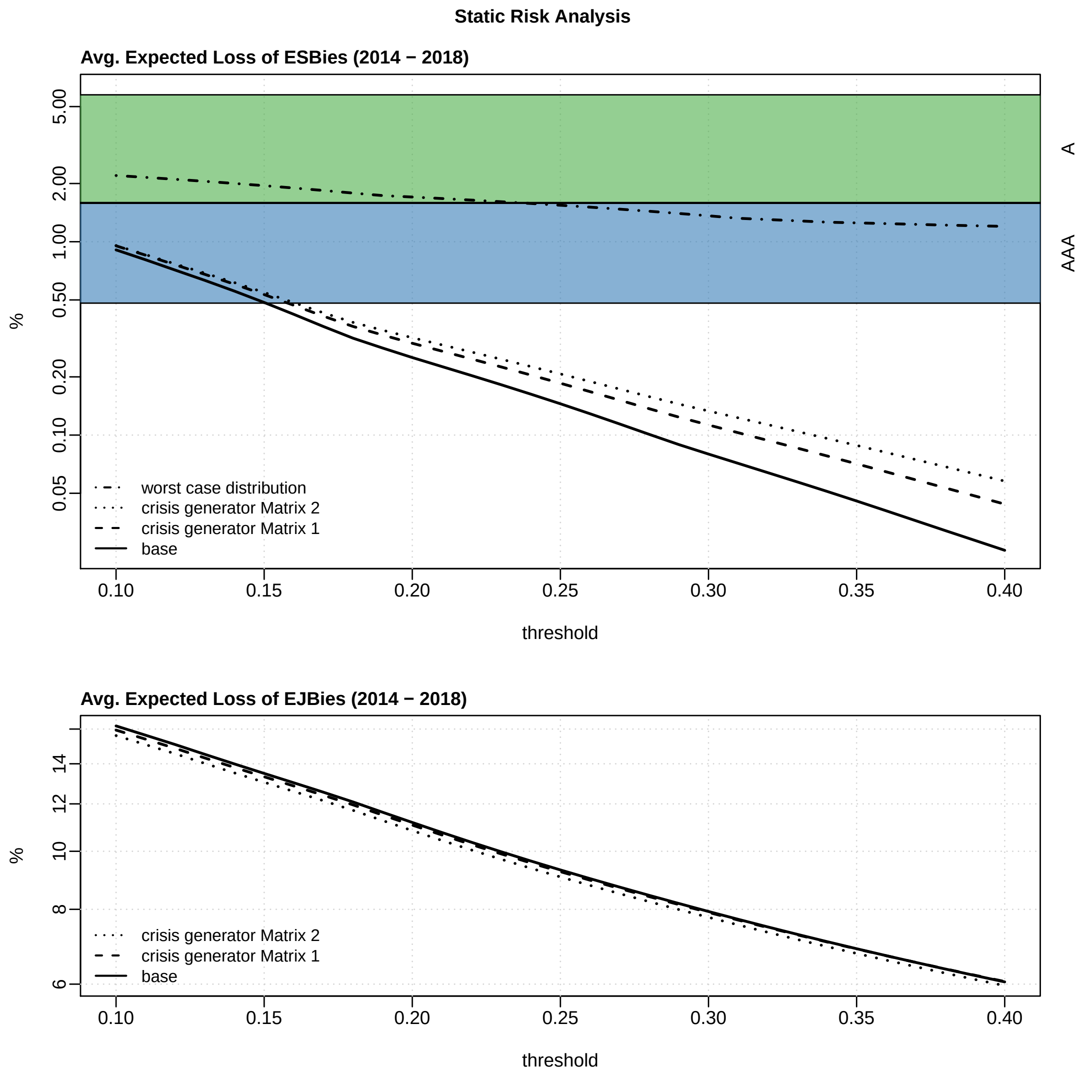

Figure 2: Average expected loss of ESBies (top) and of EJBs (bottom) for different thresholds and parameterizations (in %). Note that both graphs use a logarithmic scale on the y-axis.

6.2 Spread Trajectories of ESBies

In Figure 3 we plot trajectories of the annualized credit spread $c^{\mathrm{ESB},\kappa}$ of ESBies over the whole sample period for different levels of κ. These spreads were computed from our model by a historical simulation approach using the calibrated trajectories $\{\gamma_{s_m}\}$ and $\{X_{s_m}\}$ as input. The solid line gives the spread of an ESB with attachment point $\kappa = 0.3$ (the value proposed by Brunnermeier et al. (2017)); the colored lines correspond to different attachment points

$\kappa \in [0.2, 0.4]$. The simulated ESB spreads peak in 2009 (the height of the financial crisis) and in the period 2011–2013 (the height of the European sovereign debt crisis). We see that the attachment point has a large impact on the volatility of ESB spreads. In particular for κ close to 0.2 spreads are very volatile; for $\kappa > 0.3$ on the other hand spread fluctuations are quite small.

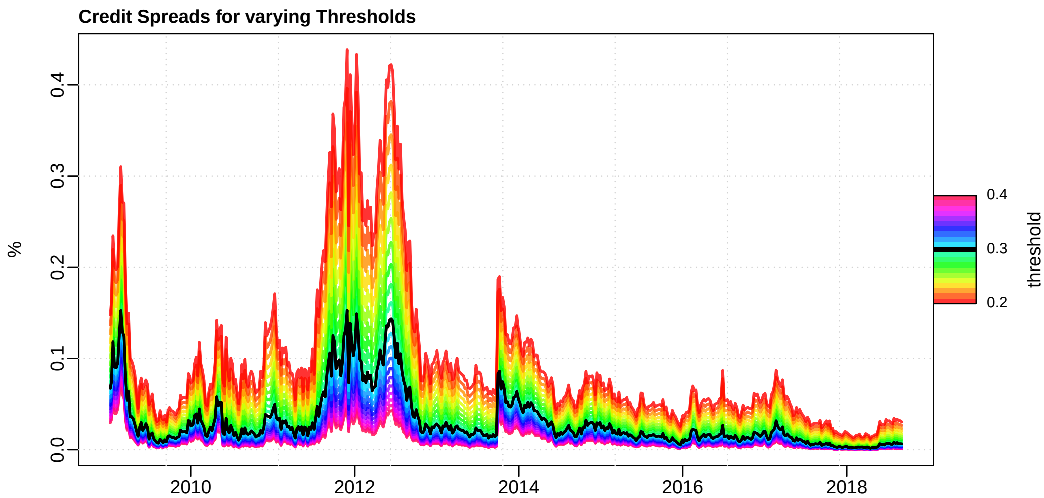

Figure 3: Spread trajectories of ESBies with varying threshold levels. The solid black line represents the reference threshold of 0.3.

6.3 Market risk analysis via scenarios

In this section, we analyze how the risk-neutral loss probability $\mathbb{Q}(L_T > \kappa)$ of ESBies is affected by changes in the underlying risk factors $X_0, \boldsymbol{\gamma}_0$ and $\boldsymbol{L}_0$. In mathematical terms, we consider the function

$$\kappa \mapsto p^{\kappa}(X_0, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,;\, Q, \boldsymbol{\mu}) := \mathbb{Q}(L_T > \kappa \mid X_0, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,;\, Q, \boldsymbol{\mu}).$$

We consider different sets of risk factor changes or *scenarios*. First we study scenarios which are included in the support of the default model from Section 2, such as a change in the state of X. Moreover, we consider several *contagion scenarios* where, in reaction to a default of Italy,[10] the market becomes more risk averse and changes its perception of the state of X and the parameter set used to value ESBies. In fact, investors on markets for (quasi) safe assets frequently change their expectations in reaction to adverse events, putting more mass on bad outcomes; see for instance Gennaioli et al. (2012).

Non-contagion scenarios. In the left panel of Figure 4, we graph the function p^{κ} on a log-scale using the parameters of the base scenarios and the calibrated values of $\boldsymbol{\gamma}$ and X for September 3, 2018 (the last observation date in our sample). The full circles give the loss probability for varying κ for the *base scenario*, where the chain is in state one (the

[10] We consider a default of Italy since on the day we used for this analysis (September 3, 2018, the last observation date in our sample) Italy had the highest CDS spread of all major euro area economies. A default of another major 'risky' euro area sovereign would yield similar results.

good economic state) and no euro area member is in default (in mathematical terms this is the function $\kappa \mapsto p^\kappa(1, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,; Q, \boldsymbol{\mu}))$. Moreover, we consider four types of changes in the underlying risk factors:

(i) the scenario where all hazard rates experience an upward jump of 10%, that is we plot the function $\kappa \mapsto p^\kappa(1, \boldsymbol{\gamma}_0 \times 1.1, \boldsymbol{L}_0\,; Q, \boldsymbol{\mu})$;

(ii) the scenario where the economy moves to a light recession, corresponding to the function $\kappa \mapsto p^\kappa(2, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,; Q, \boldsymbol{\mu})$;

(iii) the scenario where the economy moves to a severe recession ($\kappa \mapsto p^\kappa(3, \boldsymbol{\gamma}_0, \boldsymbol{L}_0\,; Q, \boldsymbol{\mu})$);

(iv) the scenario where Italy defaults with random LGD δ^{ITA}. We assume that δ^{ITA} is beta distributed with mean 0.5, i.e. the loss vector at $t = 0$ takes the form $\boldsymbol{L}_0 = (0, \ldots, 0, \delta^{\mathrm{ITA}}, 0, \ldots, 0)$, but all other risk factors stay unchanged.

The horizontal dashed lines correspond to the risk-neutral five year default probabilities of Germany, Belgium and Ireland under the base parametrization.

Inspection of the left panel of Figure 4 shows first that a change in the hazard rates has only a small impact on the loss probability of ESBies. The default of a major euro area sovereign such as Italy has a stronger effect, but for $\kappa > 0.25$, the loss probability remains small even after a major default. The most important risk factor changes are clearly changes in the state of the economy. For instance, for $\kappa = 0.3$ the loss probability of an ESB in state three is slightly larger than the risk-neutral default probability of Belgium, whereas in state one the loss probability is considerably smaller than the risk-neutral default probability of Germany. Second, we observe that the threshold probabilities are decreasing in κ roughly at an exponential rate, similarly as the expected loss does. In fact, the loss probability is quite sensitive with respect to the choice of the attachment point (to see this, one may compare the values of p^κ for $\kappa = 0.35$ and $\kappa = 0.3$ in scenario (iii)).

Contagion scenarios. In the right panel of Figure 4 we graph the function p^κ (again on a log-scale) for the base parametrization and for three different contagion scenarios, namely

(i) the case where Italy defaults and where, as a reaction, X jumps to state two (mild recession);

(ii) the case where Italy defaults and where, as a reaction, X jumps to state three (strong recession);

(iii) the case where Italy defaults and where, as a reaction, X jumps to state three and the market uses the crisis parametrization two (instead of the base parameter set). This scenario is motivated by the observation that, in the subprime crisis, investors used much more conservative assumptions for default dependence than before the crisis, see for instance Brigo et al. (2010) for details.

We see that, for an attachment point $\kappa \leq 0.3$, the change in the loss probability caused by one of the contagion scenarios is quite substantial. For instance, in the extreme scenario (iii), the risk-neutral loss probability is of the order of 5%. For attachment points $\kappa > 0.35$ the impact is less drastic. However, under scenario (iii), even for $\kappa = 0.35$ we get a risk-neutral threshold probability of around 2%, which is definitely non-negligible for a safe asset. This is in stark contrast to the analysis of the expected loss in Section 6.1, where ESBies appeared 'safe' already for $\kappa > 0.15$.

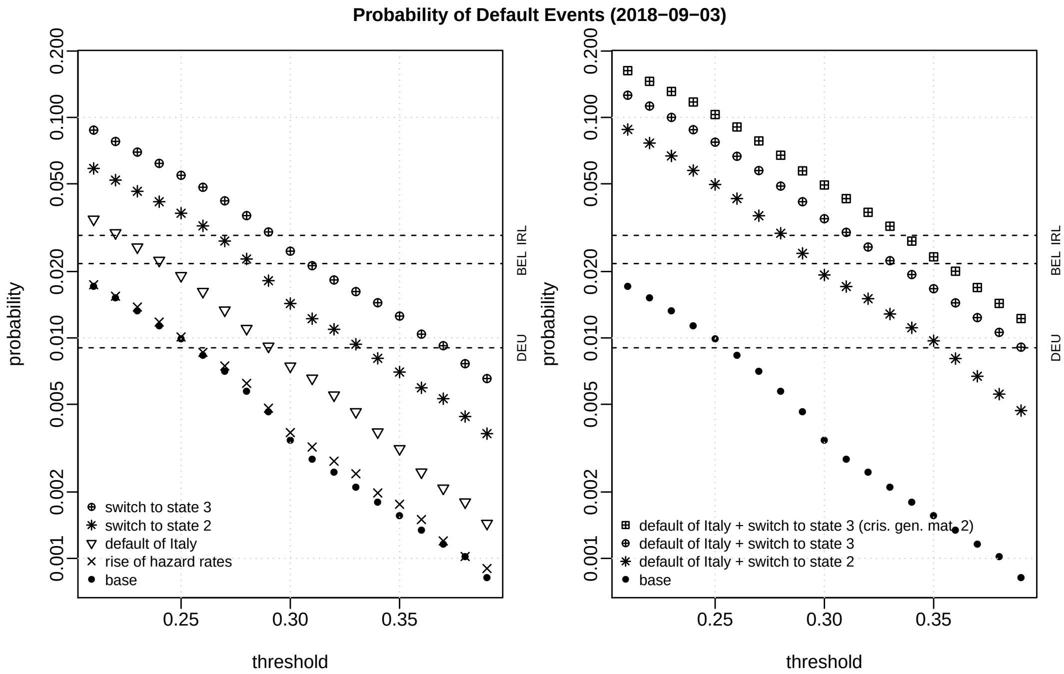

Figure 4: Loss probability of ESBies for different κ and various scenarios. Note that the plot uses a logarithmic scale on the y-axis.

6.4 Market Risk Analysis via Loss Distributions

So far we were concerned with the *value* of ESBies and EJBies in different scenarios; values were computed using the risk-neutral measure $\mathbb{Q}$, so that model parameters were derived via calibration. For computing measures of the market risk of ESBies on the other hand we have to simulate their loss distribution under the historical measure $\mathbb{P}$, so that we need to estimate the $\mathbb{P}$ dynamics of X and $\boldsymbol{\gamma}$ using statistical methods. This issue is addressed next.

EM estimation of hazard rate dynamics. In this section, we report the results of an empirical study where a model of the form (2.1) is estimated from the calibrated hazard rates of the euro area countries (the trajectories $\{\gamma_{s_m}\}$ generated in the calibration procedure of Section 4). Here we assume that the trajectory of the Markov chain is not directly observable; rather, the available information is carried by the filtration $\mathbb{F}^{\boldsymbol{\gamma}} = (\mathcal{F}_t^{\boldsymbol{\gamma}})_{t\geq 0}$ generated by the hazard rates process $\boldsymbol{\gamma}$. This assumption is motivated by the fact that the calibration of the trajectory $\{X_{s_m}\}$ in Section 4 is quite sensitive with respect to the chosen model parameters, whereas the calibration of $\{\boldsymbol{\gamma}_{s_m}\}$ is very robust (essentially due to the close connection between hazard rates and one-year CDS spreads).

Using stochastic filtering and a version of the EM algorithm adapted to our setting, we obtain the filtered and smoothed estimate for the trajectory of X, an estimate of the generator matrix of X and of country-specific parameters such as mean reversion levels and speed, all under the real-world measure $\mathbb{P}$. In the EM algorithm we use *robust* filtering techniques which

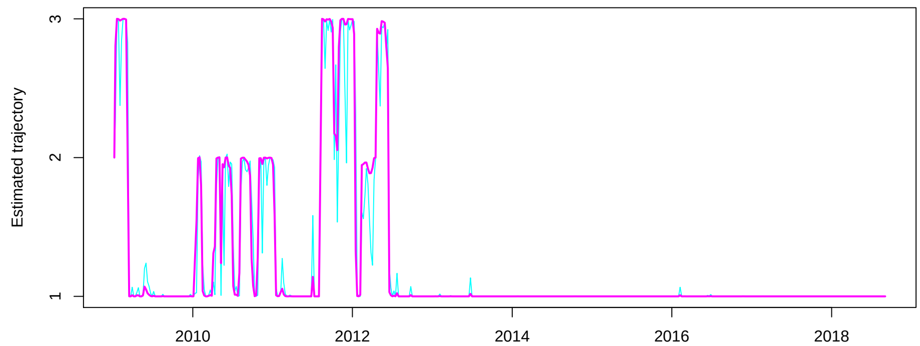

Figure 5: Filtered $(E^{\mathbb{P}}(X_t|\mathcal{F}_t^{\gamma}))$ and smoothed $(E^{\mathbb{P}}(X_t|\mathcal{F}_T^{\gamma}))$ estimates of the Markov chain trajectory.

perform well in a situation where observations are only approximately of the form (2.1). For further details on the methodology see Elliott (1993) or Damian, Eksi-Altay and Frey (2018).

We consider $K = 3$ possible states of X, corresponding to a expansionary regime, a light recession and a strong recession, respectively. The EM estimates for the generator matrix Q of X are given in Tables 8 and 9 in Appendix B, together with country-specific parameters such as mean reversion speed and levels. Note that we do not estimate the volatility, but we work with quadratic variation instead.[11] Overall the estimates appear reasonable. In particular, the estimated mean reversions levels for most countries respect the ordering $\mu^j(1) < \mu^j(2) < \mu^j(3)$, supporting the interpretation of the states of X.[12] As expected, for any given state of the economy the estimated levels are lowest for the stronger euro area countries. In Figure 5, we give a trajectory of the filtered and the smoothed state of X, that is we plot the trajectories $t \mapsto E^{\mathbb{P}}(X_t \mid \mathcal{F}_t^{\gamma})$ and $t \mapsto E^{\mathbb{P}}(X_t \mid \mathcal{F}_T^{\gamma})$. These results show that the proposed model describes the qualitative properties of euro area credit spreads and, in particular, the co-movement of spread levels of the weaker euro-area members reasonably well. The frequent transitions in and out of the middle state are not surprising, given that this state reflects a situation where only a few countries experience a rise in default intensities.

Measures of market risk. We use two popular risk measures, Value at Risk (VaR_α) and Expected Shortfall (ES_α) at confidence level α, to study the tail of the loss distribution of ESBies over a horizon of three months. Denote by $\boldsymbol{\gamma}_0$ and X_0 the calibrated hazard rates and the state of X for September 3, 2018. We generate $N = 100\,000$ realizations of the hazard rates and the Markov chain with initial values $\boldsymbol{\gamma}_0$ and X_0 over a three-month horizon, using the $\mathbb{P}$-parameters estimated in the previous paragraph, and we index the simulation outcome

[11] In order to robustify the EM estimation procedure, we scale the quadratic variation of the strong euro area countries slightly upward.

[12] For Spain and Portugal, the highest mean reversion level is estimated for state 2, which is probably due to the idiosyncratic behavior these two countries, particularly Portugal, exhibit in the first months of 2012.

by $i \in \{1, \ldots, N\}$. We then compute the corresponding *relative loss*

$$R^{\kappa,i} := 1 - \frac{h^{\mathrm{ESB},\kappa}(0.25, X_{0.25}^{(i)}, \boldsymbol{\gamma}_{0.25}^{(i)}, \boldsymbol{L}_0)}{h^{\mathrm{ESB},\kappa}(0, X_0, \boldsymbol{\gamma}_0, \boldsymbol{L}_0)}.$$

VaR and expected shortfall are then computed from the empirical distribution of the sampled relative losses $\{R^{\kappa,i}, i = 1, \ldots, N\}$, see McNeil et al. (2015, Section 9.2.6) for details.

In Figure 6 summarizes our analysis. We plot estimates of VaR_α (left) and of ES_α (right) for the three-month distribution of negative ESB-returns for different κ and confidence levels $\alpha = 0.95$ (points) and $\alpha = 0.99$ (crosses). We see that both risk measure estimates decrease approximately at an exponential rate in κ. The horizontal lines in each plot represent the 95% and 99% level of the corresponding risk measure estimate for a German zero coupon bond. We observe that both the VaR_α and the ES_α of ESBies with $\kappa \geq 0.2$ are considerably smaller than the German benchmark. We conclude that, from the viewpoint of a standard market risk analysis, ESBies appear safe and that very low risk capital levels are required to back these products.

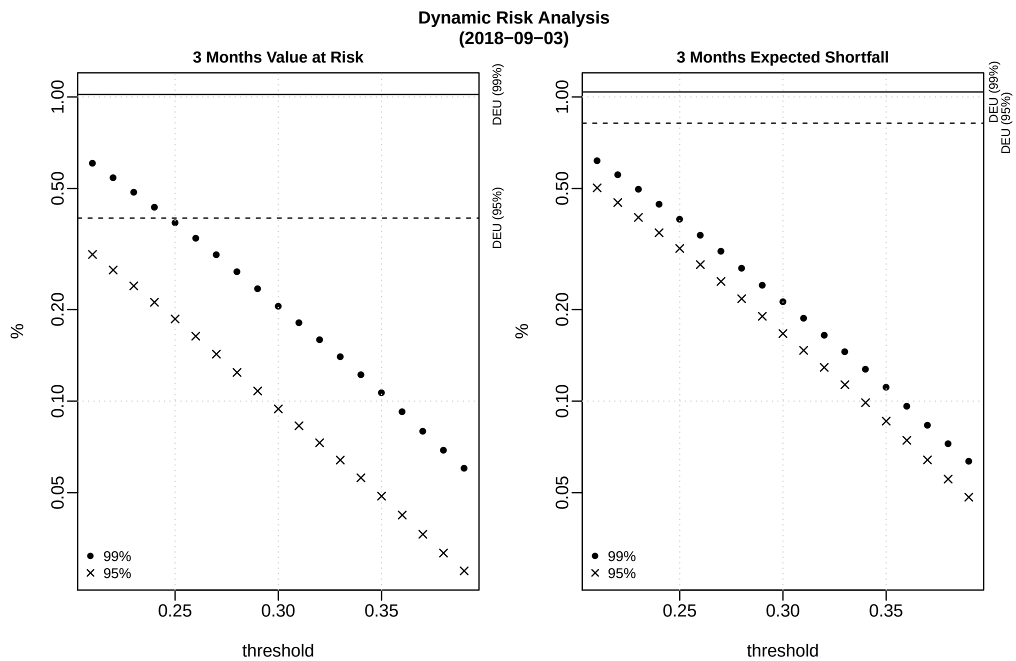

Figure 6: Risk measure estimates VaR_α (left) and ES_α (right) for the three-month distribution of negative ESB-returns for different κ and confidence levels $\alpha \in \{0.95, 0.99\}$. Note that risk measures are given in percent and that the plot uses a logarithmic scale on the y-axis.

6.5 Summary and Policy Implications

We propose the following key conclusions from the risk analysis of ESBies. Both the static risk analysis of Section 6.1 and the investigation of the loss distribution in Section 6.4 suggest that ESBies in normal circumstances diversification works and ESBies with $\kappa > 0.25$ are indeed very safe products.[13]

[13] In fact, from the perspective of an expected loss analysis, already an attachment point $\kappa = 0.15$ might suffice to make ESBies safe.

However, considering solely the results of Section 6.1 and Section 6.4 may lead to a false sense of security. The analysis of credit spread trajectories in Section 6.2 and the scenario based analysis of Section 6.3 shows that κ needs to be chosen more conservatively in order to make ESBies robust with respect to fluctuations in the underlying risk factors or changes in the market perception of default dependence. In fact, one has to take attachment points $\kappa > 0.35$ for ESBies to be safe even in very adverse scenarios. Moreover, ESBies are most likely to generate large losses in contagion scenarios. Hence, it is important that a large-scale introduction of these products is accompanied by appropriate policy measures to limit the economic implications of a default in the euro area (and thus default contagion). Such measures include a substantial weakening of the sovereign-bank nexus; a completion of the banking and capital markets union and the creation of a European deposit insurance scheme to improve risk sharing; more flexible forms of ESM (European Stability Mechanism) lending to countries in financial difficulties and bond clauses to allow for an orderly restructuring of sovereign debt and a bail-in of private investors, see Bénassy-Quéré et al. (2018) for details. If these measures are taken, the introduction of ESBies will be a useful step to improve the financial architecture of the eurozone.

A Additional Proofs and Results

A.1 Proof of Proposition 3.1

Proof. We start by conditioning in (3.7) on $\mathcal{F}_t \vee \mathcal{F}_\infty^X$. Due to the independence of the Brownian motions $W^1, \ldots, W^J$ we have conditional independence of $\gamma^1, \ldots, \gamma^J$ given $\mathcal{F}_\infty^X$, which in turn leads to

$$E\left(\xi(X_s)\exp\left(-\int_t^s \boldsymbol{a}'\boldsymbol{\gamma}_\theta d\theta\right) e^{-\boldsymbol{u}'\boldsymbol{\gamma}_s} \mid \mathcal{F}_t \vee \mathcal{F}_\infty^X\right)$$
$$= \xi(X_s)\prod_{j=1}^J E\left(\exp\left(-\int_t^s a_j\gamma_\theta^j d\theta\right) e^{-u_j\gamma_s^j} \mid \mathcal{F}_t \vee \mathcal{F}_\infty^X\right). \tag{A.1}$$

Conditional on $\mathcal{F}_\infty^X$, the hazard rates γ^j are time-inhomogeneous affine diffusions. Standard references on affine models, such as Duffie et al. (2000), consequently give that

$$E\left(\exp\left(-\int_t^s a_j\gamma_\theta^j d\theta\right) e^{-u_j\gamma_s^j} \mid \mathcal{F}_t \vee \mathcal{F}_\infty^X\right) = \exp\left(\alpha_j(t,s\,;\,X) + \beta_j(s-t,\boldsymbol{u})\gamma_t^j\right), \tag{A.2}$$

where β_j solves (3.8) and where $\frac{d}{dt}\alpha_j(t,s\,;X) = -e^{\omega^j t}\kappa^j\mu^j(X_t)\beta_j(s-t)$ and $\alpha_j(s,s\,;X) = 0$; see for instance Duffie et al. (2000) or Section 10.6 of McNeil et al. (2015) for a proof. Integration thus gives $\alpha_j(t,s\,;X) = \int_t^s e^{\omega^j\theta}\kappa^j\mu^j(X_\theta)\beta_j(s-\theta)d\theta$. By iterated conditional expectation we hence get

$$E\left(\xi(X_s)\exp\left(-\int_t^s \boldsymbol{a}'\boldsymbol{\gamma}_\theta d\theta\right) e^{-\boldsymbol{u}'\boldsymbol{\gamma}_s} \mid \mathcal{F}_t\right)$$
$$= \exp\Big(\sum_{j=1}^J \beta_j(s-t)\gamma_t^j\Big) E\left(\xi(X_s)\exp\left(\int_t^s \bar{\mu}_{X_\theta}(\theta)d\theta\right) \mid \mathcal{F}_t\right)$$

The Feynman Kac formula for functions of the Markov chain X finally gives that

$$E\left(\xi(X_s)\exp\left(\int_t^s \bar{\mu}_{X_\theta}(\theta)d\theta\right) \mid \mathcal{F}_t\right) = v(t,X_t),$$

and hence the result. □

A.2 Proof of Proposition 5.1

Proof. First we show that π^* satisfies the constraints on the expected loss. Denote by $\boldsymbol{l}^j \in \mathbb{R}^n$ the random vector with components $l^j_1 = \cdots = l^j_{j-1} = 0$, $l^j_j = \cdots = l^j_J = 1$. Then

$$E^{\pi^*}(L^j) = \pi^*(1,\ldots,1)) + \cdots + \pi^*(\boldsymbol{l}^j) = \bar{\ell}^1 + (\bar{\ell}^2 - \bar{\ell}^1) + \cdots + \bar{\ell}^j - \bar{\ell}^{j-1} = \bar{\ell}^j \,.$$

Next we verify that the random vector $\boldsymbol{L}^*$ given in (ii) has distribution π^*. In fact, it holds for $1 \le j \le J$ that

$$P(L^1 = \cdots = L^{j-1} = 0, L^j = \cdots = L^J = 1) = P(1 - \bar{\ell}^j < U \le 1 - \bar{\ell}^{j-1}) = \bar{\ell}^j - \bar{\ell}^{j-1} \,.$$

Hence, under π^*, the random variables $L^1, \ldots, L^J$ can be represented as increasing functions of a single random variable, so that they are comonotonic by definition.

To verify that π^* is the worst case distribution we show that it maximizes the value of EJBies. Since $\boldsymbol{L}^*$ has distribution π^*, we have to show that for any random vector $(L^1, \ldots, L^J)' \in [0,1]^J$ with $E(L^j) = \bar{\ell}^j$, $1 \le j \le J$, and any $\kappa > 0$,

$$E\Big(\big(\sum_{j=1}^J w^j L^j - \kappa\big)^+\Big) \le E\Big(\big(\sum_{j=1}^J w^j L^*_j - \kappa\big)^+\Big). \tag{A.3}$$

(We may use call options instead of put options in (A.3) since $E(\sum_{j=1}^J w^j L^j)$ is fixed). To establish the inequality (A.3) we use a result on stochastic orders from Bäuerle and Müller (2006). According to the equivalence ((iii) $\Leftrightarrow$ (iv)) in Theorem 2.2 of that paper, (A.3) is equivalent to the inequality

$$\mathrm{ES}_\alpha\Big(\sum_{i=1}^J w^j L^j\Big) \le \mathrm{ES}_\alpha\Big(\sum_{i=1}^J w^j L^*_j\Big) \text{ for all } \alpha \in [0,1)\,, \tag{A.4}$$

where for a generic random variable Z, $\mathrm{ES}_\alpha(Z) = \frac{1}{1-\alpha}\int_\alpha^1 q_u(Z)\,du$ gives the expected shortfall of Z at confidence level α and where $q_u(Z)$ denotes the quantile of Z at level u.

To establish (A.4) we show first that L^*_j maximizes the quantity $\mathrm{ES}_\alpha(L^j)$ over all rvs L^j with value in $[0,1]$ and expectation $E(L^j) = \bar{\ell}^j$, simultaneously for all $\alpha \in [0,1)$. In fact, the random variable L^j has to satisfy the constraints $q_u(L^j) \le 1$ (since $L^j \in [0,1]$) and $\int_0^1 q_u(L^j)\,du = \bar{\ell}^j$ (since $E(L^j) = \bar{\ell}^j$, so that

$$\mathrm{ES}_\alpha(L^j) \le \frac{1}{1-\alpha}\min\{1-\alpha, \bar{\ell}^j\} = \mathrm{ES}_\alpha(L^*_j)\,.$$

Moreover, we get from the coherence of expected shortfall that

$$\mathrm{ES}_\alpha\Big(\sum_{j=1}^J w^j L^j\Big) \le \sum_{j=1}^J w^j \mathrm{ES}_\alpha\big(L^j\big) \le \sum_{j=1}^J w^j \mathrm{ES}_\alpha\big(L^*_j\big) = \mathrm{ES}_\alpha\Big(\sum_{j=1}^J w^j L^*_j\Big)\,,$$

where the last equality follows since the random variables $L^*_1, \ldots, L^*_m$ are comonotonic. This gives inequality (A.4) and hence the result. □

A.3 Approximation of the worst-case distribution

In this section we sketch an approach for the approximation of the worst-case distribution π^* from Proposition 5.1 within our model. In line with the notation used in the proposition, we assume that sovereigns are ordered according to their credit quality. Note first that for κ^j large and σ^j small, the hazard-rate trajectory $(\gamma_t^j)_0$ is essentially determined by the trajectory of X and by the choice of the mean reversion level $\mu^j(\cdot)$ so that we concentrate on these quantities. We consider a model with $K = J+1$ states of X that correspond to the different default "clusters" under π^*. Choose some large n and define the mean reversion level $\mu^j(\cdot)$ by $\mu^1(1) = \cdots = \mu^J(1) = \frac{1}{n}$; $\mu^1(2) = \cdots = \mu^{J-1}(2) = \frac{1}{n}, \mu^J(2) = n$ and so on, in particular $\mu^1(J+1) = \cdots = \mu^J(J+1) = n$. Note that in state k, the default probability of obligor 1 to obligor $J-k+1$ is small, the default probability of obligor $J+2-k$ up to obligor J is large, that is the state corresponds to the $J+2-k$th default cluster.

Next we define the generator matrix of X. We assume that states 2 to $J+1$ are absorbing so that $q_{ik} = 0$ for $2 \le i \le J+1$ and all k. Define probabilities $p_1, \dots, p_{J+1}$ by $p_1 = 1 - \bar{l}^J$, $p_k = \bar{\ell}^{J+2-k} - \bar{\ell}^{J+1-k}$ for $2 \le k \le J$, and finally $p_{J+1} = \bar{\ell}^1$, that is p_k corresponds to the probability of the $J+2-k$th default cluster under π^*. Since states $2, \dots, J+1$ are absorbing we get for any valid choice for the first row of Q that $\mathbb{Q}(X_T = 1) = e^{q_{11}T}$ and

$$\mathbb{Q}(X_T = k) = (1 - e^{q_{11}T})\frac{q_{1k}}{-q_{11}}, \quad k = 2, \dots, J+1\,.$$

(recall $q_{11} = -\sum_{k=2}^{J+1} q_{1k}$). We want to choose q_{1k}, $2 \le k \le J+1$ so that $\mathbb{Q}(X_T = k) = p_k$, $1 \le k \le J+1$. This gives for the transition intensities that

$$q_{11} = \frac{1}{T}\ln p_1 \text{ and } q_{1k} = -p_k \frac{q_{11}}{1-p_1}\,, k = 2, \dots, J+1\,. \tag{A.5}$$

Since $\sum_{k=1}^{J+1} p_k = 1$, we get that $q_{11} = -\sum_{k=2}^{K+1} q_{1k}$ so that (A.5) defines indeed a valid generator matrix. Moreover, for $n \to \infty$, $\kappa^j \to \infty$ and $\sigma^j \to 0$,

$$\mathbb{Q}\big(Y_1 = \cdots = Y_{J-k+1}) = 0, Y_{J-k+2} = \dots Y_J = 1\big)$$

converges to $\mathbb{Q}(X_T = k) = p_k$ which gives the result by definition of the p_k.

B Details on Calibration

B.1 Data

In Table 4 below we present summary statistics of the data we use in the model calibration.

B.2 Methodology

In order to determine the parameters (Θ^j, σ^j), $1 \le j \le J$, the generator matrix Q and the realised trajectories $\{\gamma_{s_m}\}$ and $\{X_{s_m}\}$, we use an iterative approach which is compactly summarized in Algorithm 1 below. We set $\mathbf{\Theta} = (\Theta^1, \dots, \Theta^J)$ and we use $\{\gamma_{t_m}\}^{(i)}$, $\{X_{t_m}\}^{(i)}$ and $(\Theta^j)^{(i)}$ to denote the i-th estimate of the distinct variables within the iteration.

Yrs.	AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
	AA	AA	AAA	A	AA	AA	A	BBB	AAA	BBB
					Panel A: Mean					
1	31.071	44.063	12.819	113.637	13.273	26.590	204.380	115.934	20.269	307.199
2	38.341	54.593	16.700	138.474	17.780	35.106	220.752	143.971	25.232	346.413
3	45.016	66.843	21.918	153.769	22.148	44.961	224.791	165.881	30.413	352.693
4	54.657	77.339	29.151	165.480	28.219	56.684	223.055	181.786	38.267	354.438
5	61.675	85.437	34.562	174.373	33.048	65.906	222.409	193.090	43.902	359.586
					Panel B: Standard Deviation					
1	38.589	59.330	12.418	113.398	12.932	30.303	309.597	110.190	21.720	436.057
2	42.347	67.134	14.520	129.415	14.257	34.529	308.774	113.689	23.782	449.052
3	44.800	74.987	17.157	131.520	15.065	39.920	293.590	115.705	24.563	399.150
4	48.974	76.066	21.330	130.483	16.769	45.769	264.432	113.967	27.595	352.837
5	51.023	76.099	24.114	129.470	17.407	49.149	244.132	112.625	29.439	324.968
					Panel C: Minimum					
1	4.080	3.840	2.920	10.450	2.250	3.550	7.830	21.620	3.120	12.480
2	6.190	7.020	3.980	18.900	3.800	6.570	12.330	33.880	4.840	28.670
3	7.820	9.430	6.230	25.050	6.040	9.620	15.860	48.990	7.230	40.650
4	9.890	11.910	8.330	30.730	10.290	12.630	19.670	56.490	9.140	47.110
5	13.270	16.480	9.510	37.230	13.020	17.400	23.970	59.830	11.240	47.230
					Panel D: Maximum					
1	259.960	301.620	74.840	489.430	66.530	160.660	1629.340	619.540	110.870	2598.930
2	267.440	337.600	81.080	608.330	74.600	177.440	1614.480	591.030	125.040	2494.690
3	269.490	375.700	90.350	619.920	82.550	198.200	1572.800	581.050	130.650	2102.190
4	271.430	379.090	108.250	622.220	90.460	222.510	1419.750	575.930	132.710	1846.700
5	272.180	380.940	119.060	624.290	95.000	237.300	1318.590	573.030	136.960	1802.360

Table 4: Summary statistics of CDS spreads (in bp).

Algorithm 1: Detailed description of calibration step

Data: Market CDS spreads for maturities $u \in \mathcal{T}$ for each sovereign $1 \leq j \leq J$

Result: Estimates for $\{\boldsymbol{\gamma}_{s_m}\}$, $\{X_{s_m}\}$ and $\boldsymbol{\Theta}$

Initialization for $\{\boldsymbol{\gamma}_{s_m}\}^{(0)}$, $\{X_{s_m}\}^{(0)}$, $(\boldsymbol{\Theta})^{(0)}$ and $Q^{(0)}$

$i = 0$

while $\sum_{j=1}^{J} \sum_{m=0}^{M} l^j \left((\gamma^j_{s_m})^{(i)}, (\Theta^j)^{(i)}, (\sigma^j)^{(i)}, Q^{(i)}, X^{(i)}_{s_m} \right) \geq \epsilon$ **do**

  for $j \leftarrow 1$ **to** J **do**

    for $m \leftarrow 0$ **to** M **do**

      $(\gamma^j_{s_m})^{(i+1)} = \arg\min_{\gamma} l^j_{s_m}(\gamma, (\Theta^j)^{(i)}, (\sigma^j)^{(i)}, Q^{(i)}, X^{(i)}_{s_m})$

    end

    Estimate $(\sigma^j)^{(i+1)}$ based on the quadratic variation of $(\gamma^j)^{(i+1)}$

  end

  for $m \leftarrow 0$ **to** M **do**

    $X^{(i+1)}_{s_m} = \arg\min_x \sum_{j=1}^{J} l^j_{s_m} \left((\gamma^j_{s_m})^{(i+1)}, (\Theta^j)^{(i)}, (\sigma^j)^{(i+1)}, Q^{(i)}, x) \right)$

  end

  Estimate $Q^{(i+1)}$ via MLE based on $X^{(i+1)}$

  for $j \leftarrow 1$ **to** J **do**

    $(\Theta^j)^{(i+1)} = \arg\min_{\Theta} \sum_{m=0}^{M} l^j_{s_m} \left((\gamma^j_{s_m})^{(i+1)}, \Theta, (\sigma^j)^{(i+1)}, Q^{(i+1)} X^{(i+1)}_{s_m}) \right)$

  end

  Set $i \leftarrow i + 1$

end

The assumption of conditionally independent defaults substantially facilitates the calibration procedure: given an estimate for Q and $\{X_{s_m}\}$, estimation of $\{\gamma^j_{t_m}\}$ and of the parameter vector Θ^j can be done independently for each sovereign j. We initiate the calibration by applying k-means clustering on the relevant CDS spreads to get an estimate for $X^{(0)}$. For small maturities T it holds that $\widehat{\text{cds}}^j_T \approx \delta^j(X)\gamma^j$. We use this approximation along with the initial estimate $X^{(0)}$ to get an estimate for $(\gamma^j)^{(0)}$ and we consequently solve the optimization problem of line 15 in Algorithm 1 to obtain the initial value $\mathbf{\Theta}^{(0)}$. To compute the estimates for σ^j we use that the quadratic variation of γ^j satisfies

$$[\gamma^j, \gamma^j]_t = (\sigma^j)^2 \int_0^t \gamma^j_s ds,$$

and we approximate the integral with Riemann sums. For a given (estimated) realisation of the Markov chain we use the standard MLE estimator for continuous-time Markov chains to get an estimate of Q.

The main numerical challenge in the application of Algorithm 1 is to solve the optimization problem

$$\min_{\Theta} \sum_{m=0}^{M} l^j \left((\gamma^j_{s_m})^{(i+1)}, \Theta, (\sigma^j)^{(i+1)}, Q, X^{(i+1)}_{s_m}) \right). \tag{B.1}$$

We impose the restriction that all parameters are non-negative and, for regularization purposes, we set the lower bound of the mean-reversion speed κ^j to 0.1 for all j. During the first iteration of Algorithm 1 we employ the implementation of Johnson (2019) of an algorithm for constrained optimization as presented in Runarsson and Yao (2005). The algorithm uses heuristics to escape local optima. In order to refine the estimation, in the subsequent calibration steps (i.e. for steps $i > 1$) we use the local optimizer of Powell (1994), which provides a derivative-free optimization method based on linear approximations of the target function. Again we resort to the implementation of Johnson (2019). The optimization problem linked to Θ^j is computationally expensive, as the re-valuation of $l^j_{t_m}$ requires frequent solutions of the ODE system (3.9). Here we resort to the `C++` library `odeint` of Ahnert and Mulansky (2011) which is part of the `boost` library collection.

After successful convergence of Algorithm 1 we perform a final refinement step, in which we keep all input variables except Θ^j, $1 \leq j \leq J$ fixed. We resort to the package `ROI` of Theußl, Schwendinger and Hornik (2017). The architecture of `ROI` enables us to employ numerous solvers for the optimization problem (B.1). For each sovereign, the parameters reported in Table 2 correspond to the solver which returns the smallest value of the relevant loss function.

B.3 Results

State	AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
1	0.55	0.55	0.50	0.55	0.50	0.50	0.55	0.50	0.50	0.55
2	0.55	0.55	0.50	0.55	0.50	0.50	0.55	0.50	0.50	0.55
3	0.65	0.65	0.60	0.65	0.60	0.60	0.65	0.60	0.60	0.65

Table 5: Fixed conditional means of LGDs for different sovereigns and varying states.

The following figure illustrates the quality of the model fit for two different sovereigns.

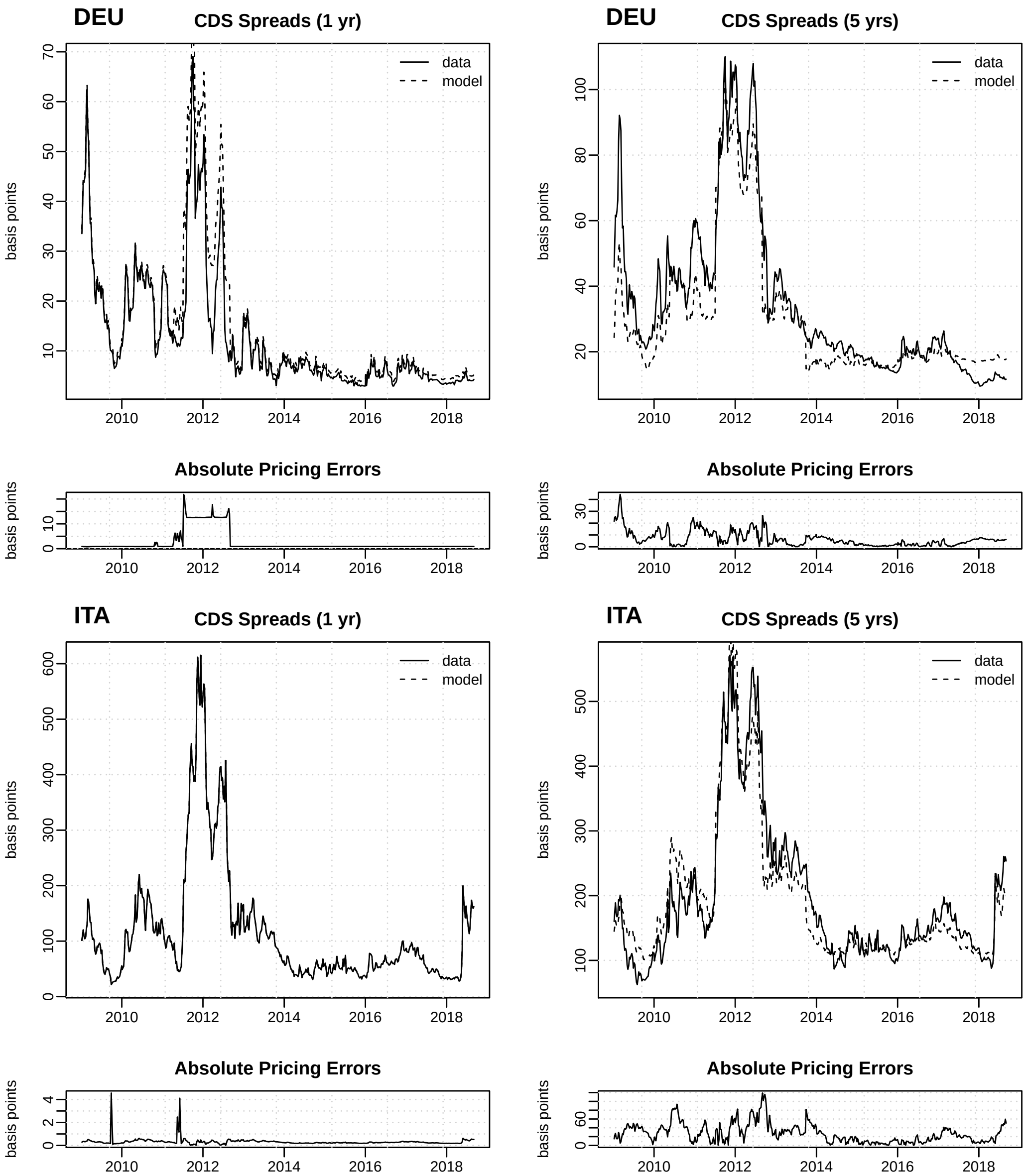

Figure 7: Time series plots of market CDS spreads against model values. The solid (dashed) lines correspond to the market (model) values of the distinct CDS spreads.

B.4 Parameters used in Risk Analysis

AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
0.04	0.04	0.29	0.12	0.02	0.20	0.03	0.18	0.07	0.01

Table 6: Portfolio weights of ESBies and EJBies, based on proportion of sovereigns on euro area GDP as of 2018.

	$\widetilde{Q}_1$			$\widetilde{Q}_2$		
	State 1	State 2	State 3	State 1	State 2	State 3
State 1 (expansion)	−0.1421	0.1421	0.0000	−0.1421	0.1421	0.0000
State 2 (mild recession)	0.5843	−0.8685	0.2843	0.5843	−0.7685	0.1843
State 3 (strong recession)	0.0000	1.4444	−1.4444	0.0000	1.4444	−1.4444

Table 7: Generator matrices $\widetilde{Q}_1$ and $\widetilde{Q}_2$ for crisis scenarios.

B.5 Results of EM Estimation

Param.	AUT	BEL	DEU	ESP	FIN	FRA	IRL	ITA	NLD	PRT
$\mu(1)$	0.0023	0.0016	0.0012	0.0013	0.0029	0.0220	0.0329	0.0095	0.0136	0.0027
$\mu(2)$	0.0103	0.0054	0.0013	0.0016	0.0196	0.1391	0.1375	0.0408	0.1231	0.0099
$\mu(3)$	0.0144	0.0120	0.0116	0.0080	0.0346	0.1219	0.0941	0.0698	0.1245	0.0192
κ	6.9584	6.2427	3.1193	6.3241	5.5822	3.4718	1.5879	3.2518	2.0640	8.4694

Table 8: Estimation results: parameters of hazard rate dynamics.

	State 1	State 2	State 3
State 1 (expansion)	−0.9033	0.9033	0.0000
State 2 (mild recession)	5.9877	−10.4716	4.4839
State 3 (strong recession)	4.3316	1.8569	−6.1885

Table 9: Estimation results: generator matrix Q of X.

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