On strong approximation of SDEs with non-globally Lipschitz continuous coefficients

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We consider the problem of strong approximation of the solution of a stochastic differential equation (SDE) at the final time based on finitely many evaluations of the driving Brownian motion W. While the majority of results for this problem deals with equations that have globally Lipschitz continuous coefficients, such assumptions are typically not met for real world applications. In recent years a number of positive results for this problem has been established under substantially weaker assumptions on the coefficients: new types of algorithms have been constructed that are easy to implement and still achieve a polynomial rate of convergence under these weaker assumptions.

In the first part of the talk we present negative results for this problem. We show that there exist SDEs with bounded smooth coefficients such that their solutions can not be approximated by means of any kind of adaptive method with a polynomial rate of convergence. Even worse, we show that for any sequence $(a_n)_{n\in\mathbb{N}} \subset (0,\infty)$, which may converge to zero arbitrarily slowly, there exists an SDE with bounded smooth coefficients such that no approximation method based on n adaptively chosen evaluations of W on average can achieve a smaller absolute mean error than the given number a_n.

In the second part of the talk we present positive results in the case when the drift coefficient may have discontinuities in space. We show that for scalar SDEs with a piecewise Lipschitz continuous drift coefficient and a Lipschitz continuous diffusion coefficient that is non-zero at the discontinuity points of the drift coefficient the Euler scheme achieves an L_p-error rate of at least $1/2$ for all $p \in [1,\infty)$. So far only an L_p-error rate of at least $1/(2p)-$ was known in the literature for the Euler scheme in this setting. We furthermore present a numerical method, which achieves an L_p-error rate of at least $3/4$ for all $p \in [1,\infty)$ under additional piecewise regularity assumptions on the coefficients.

The talk is based on joint work with Arnulf Jentzen (ETH Zurich) and Thomas Müller-Gronbach (University of Passau).